

VISUALIZATION OF THE MULTI-ATTRIBUTE DESIGN SPACE

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1. Introduction

A multi-attribute design procedure results in relatively large number of Pareto optimal designs. Dimensionality of the multi-dimensional design space depends on number of attributes. In different ship concept design models developed at University of Zagreb 5-14 attributes were used as measures of merit. Since every design is also defined by a set of parameters it is necessary to establish relations between design parameters and design attributes. It means that in practice dimensionality of the problem equals the sum of number of design variables (parameters) and the number of design attributes. Obviously, multi-dimensional space can not be visualized directly. Designer is confronted with the problem of visualizing anything from 10 to 25 –dimensional space. In order to visualize multi dimensional space a special procedure is devised.

2. Multi-attribute design procedure

In order to study the problem of visualization use is made of the multi-attribute design procedure as applied to the problem of concept design of the fast patrol and SAR (search and rescue) vessels as shown in [Grubišić and Begović 2003] where appropriate mathematical model and design parameters and attributes are presented.

The procedure is based on the concept of non-dominance. It means that it is not possible to improve on any attribute of the non-dominated design without the loss in some of the other attributes. Pareto frontier consists of all non-dominated designs. The unique optimal design composed from the best attribute values reached by the Pareto frontier, is a fictitious "ideal" design usually designated as Utopia. Since this attribute values belong to different designs Utopia is infeasible. Non-dominated designs have some useful characteristics:

- Non-dominance is not influenced by subjectivity.
- Relation of design parameters and design attributes is established for the best possible designs
- Every design may be related to the Utopia by the L_1 , L_2 or L_∞ metrics.
- The preferred design among all non-dominated ones is selected *ex post* by a separate procedure that may include subjectivity.

The design procedure consists of several levels as presented in [Figure 1]. In the case at hand a set of 10 design parameters is generated at random using adaptive Monte Carlo method. These parameters are subject to parameter constraints before entering the design model. If design is feasible the model yields a set of 7 design attributes that are subsequently subject to attribute constraints. Surviving feasible designs are further filtered to obtain non-dominated ones defining Pareto-optimal frontier. In the example presented a set of 3372 non-dominated designs were found. Each of the 3372 Pareto-optimal design is defined by the values of 10 parameters and 7 attributes, therefore visualization is dealing with 17 dimensions.

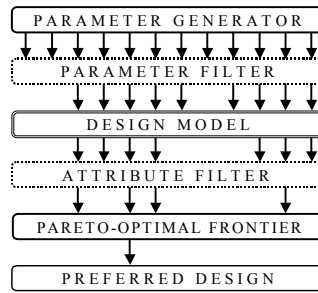









Figure 1. Multi-attribute design procedure

3. Visualization

The purpose of visualization is insight, therefore, the theory of perception should help us to visualize abstract data [Bertin 1983]. Visualization of the multi-dimensional space is difficult for humans. Development of computer graphics opened new possibilities that are not possible using paper bound graphics [Green]. In addition to the limitations of attention and memory, humans often perceive data presentation and coding as non-proportional, confusing and difficult to interpret. According to Bertin data presentation may use the following graphical variables:

Table 1. Graphical variables (after Bertin)

	Planar
	Size
	Brightness
	Texture
	Color (Hue)
	Orientation
	Shape

Green added more variables [Green] made possible by the computer graphics on CRT or LCD screen. Since paper presentation is still important we will limit discussion to the Bertin's set only. First two variables are planar and are the most important since they are directly represented in retinal image. Other six variables are retinal and must be interpreted in some way. Here the issue is presentation of some magnitude (like distance from the Utopia), therefore quantitative distinction is needed. Only the size and partially brightness and colour (hue) are useful in that respect, while the orientation and shapes are the worst since they are not associative. It is difficult to visualize 4 or more components of the image since visualization requires visual integration across the image. Therefore, we are left with 3 graphical variables, i.e. 2 planar variables and one retinal variable. Size or colour may be used, since brightness variation is difficult to represent on paper.

Number of levels of absolute judgment is limited to about 5 [Ward]. Relative detection is more reliable than absolute one. Our capacity to distinguish and keep in immediate memory is limited to 4 - 7 items. It may be concluded that a safe approach would be to use 2 planar variables and one retinal and increment it in 5 strata by changing size or colour. Two problems have to be addressed: graphical coding and stratification.

4. Graphical coding

Development of multi-attribute design methods at University of Zagreb began in 1987 and at first used printed alpha-numerical presentation limited by the technology of that time [Žanić et al. 1992]. Different symbols from the Courier font were used to suggest variable size and so suggest distance from Utopia [Table 2]. Although it may be considered crude this is still basically sound and very efficient approach. It is convenient to program it in FORTRAN since design model and optimization

procedure are programmed that way. Principal disadvantage of this approach is the limited number of columns and rows that can be represented on screen, resulting in coarse graduation of attribute values.

Table 2. Font symbols as a graphical code

n	1	2	3	4	5
Courier New	.	+	*	o	o

Finer representation is possible by using graphical capabilities of spreadsheet programs (e.g. MS EXCEL). Procedure may be integrated in to the host program or an intermediate file may serve as an input to the separate visualization software.

Colour may be brought in for online screen use although it is still generally not accepted for printing where shades of grey replace colour. Colour code is developed by mimicking quenching colours (to make it associative to engineers) and in shades of grey to make it suitable for black and white printer. In order to reproduce colours RGB value is given in the [Table 3].

Table 3. Colour code

SHADES OF GREY (linear)			QUENCHING COLOURS		
i	RGB	COLOUR	RGB	COLOUR	
1	255,255,255	white	255,255,255	white	
2	204,204,204	grey - 20%	255,255,153	light yellow	
3	153,153,153	grey - 40%	255,204, 0	gold	
4	102,102,102	grey - 60%	255,153, 0	light orange	
5	51, 51, 51	grey - 80%	255, 0, 0	red	
6	0, 0, 0	black	153, 51, 0	brown	

Human perception of relative magnitude of symbols is not proportional. Double intensity does not produce double sensation [Green]. Physical magnitude is related to sensation by the Steven's law:

$$\psi = k \cdot \Phi^\beta \quad (1.1)$$

,where ψ is level of sensation, Φ is physical magnitude, k is constant of proportion and β is the slope in the log-log diagram.

Physical magnitude in this case is the area of a circle. The exponent $\beta = 0.56$ was found from experiments [Ward], while k should be determined to give suitable size in relation to other elements of image.

Table 4. Size of graphical symbol

i	RENARD			FIBONACCI			LINEAR		
	$D_R = 10^{\frac{K-i}{K-1}}$			$D_F = 1+9 \cdot \frac{f_K - f_i}{f_K - 1}$			$D_L = 1+9 \cdot \frac{K-i}{K-1}$		
1	○	10	○	○	10	○	○	10	○
2	●	5,6	●	●	8,7	●	●	7,8	●
3	●	3,2	●	●	7,4	●	●	5,5	●
4	●	1,8	●	●	4,9	●	●	3,3	●
5	●	1	●	●	1	●	●	1	●

It is interesting to notice that modified Renard series (1.2) produces curve very close to the Steven's law (1.3) when reduced to diameter. Other possibilities include use of the Fibonacci series.

$$D_R = 10^{\frac{K-i}{K-1}} \quad (1.2)$$

$$D_S = 19.055 \cdot 10^{-0.56 \cdot i/2} \quad (1.3)$$

Diameters of symbols in the range of 1-10 are generated in [Table 4] for variable number of strata K . Combined usage of shape size and colour simultaneously is presented for 5 strata i.e. $K=5$. In [Table 4] use is made of the Fibonacci series where $f_K = (1,2,3,5,8,13...)$

5. Stratification of the attribute space

Different metrics, L_1 , L_2 or L_∞ , may be utilized in the multi-dimensional space. Experience in practical work suggests that L_∞ is slightly preferred although L_2 produces similar results. Non-dominated designs are stratified in proportion to their distance to the Utopia. Designs that are closest to Utopia are placed in the stratum of best designs followed by the subsequent designs in more distant strata. Two distributions are tried in which N strata are defined by pointers to the position of the list, S_i , where p is population (in this case 3372) of the Pareto frontier [Table 5]. Large number of strata is not practical since perception fails to discriminate small variation in size.

Table 5. Pointers for variable stratification for 3372 non-dominated designs

i	$S_i = p^{\frac{i}{N}}$					$S_i = p \cdot 2^{i-N}$				
	N=3	N=4	N=5	N=6	N=7	N=3	N=4	N=5	N=6	N=7
1	15	8	5	4	3	843	422	211	105	53
2	225	58	26	15	10	1686	843	422	211	105
3	3372	443	131	58	33	3372	1686	843	422	211
4		3372	664	225	104		3372	1686	843	422
5			3372	871	331			3372	1686	843
6				3372	1057				3372	1686
7					3372					3372

Experiments with variable number of strata indicate that 5 strata is practical to work with, while even 3 strata are quite satisfactory for fast work.

6. Results

Since the designs in multi-attribute approach are generated by individual trials, either at random by a adaptive Monte Carlo method, or in the more systematic way by the fractional factorial experiment, the result is a number of individual designs representing points in multi-dimensional space. Therefore the idea of viewing design space as a sort of planetarium is self evident.

Examples of the possible views into attribute space generated by the optimisation software for the fast patrol and SAR vessel design model, demonstrate the effects of selecting the number of strata and the size and colour of symbols representing individual designs.

In [Figures 2...5] the multi-dimensional space is visualised through 2-dimensional window. Therefore, for 17-dimensional space there are $16+15+14+\dots = n \cdot (n-1) / 2 = 136$ possible views (ignoring swapping of 2D axis).

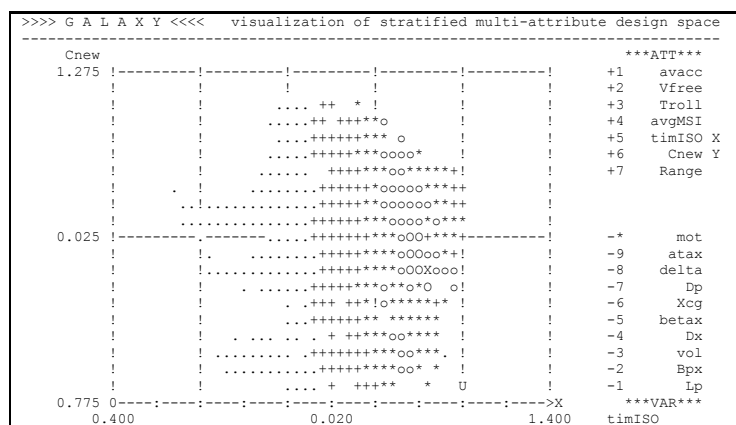


Figure 2. Alpha-numerical visualisation of the multi-attribute design space

All other attributes that are not selected as planar ones are combined to give a synthetic distance from Utopia measured by L_∞ metrics.

Visualization of the Pareto-optimal hyper-surface is concerned with the viewer (designer) located at the level of Utopia looking through 2D window distant objects in different strata.

One of the 136 possible views at the Pareto frontier is shown in [Figure 2] using font symbols with size proportional to distance [Table 2]. Attributes shown on axis deal with time the vessels are operative in variable sea state and the cost of new vessel and are only illustrative. Letter U in [Figure 2] stands for Utopia and symbol X shows position of the preferred design by the L_∞ metric. Here the problem of weighting of different attributes is not discussed since it does not influence Pareto frontier, only different preferred design may be indicated.

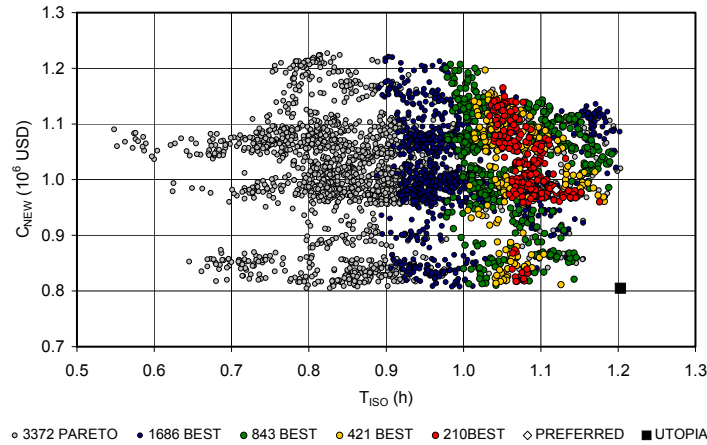


Figure 3. Stratified Pareto-optimal hyper-surface with variable colour

The same view as in [Figure 2] is presented in [Figure 3] but this time using MS-EXCEL and with variable colour for different strata. According to [Lewandowsky and Spence 1989] colour is best used to present stratified data. Symbol size was not varied in order to test colour grading only. Legend of colours is included in the figure. It may be said that this is much better view although it contains the same data.

Further examples of the possible views into attribute space generated by the optimisation software for the patrol and SAR boat design model, demonstrate the effects of varying symbol size and colour. In the [Figure 4] the same Pareto frontier is depicted by the variable diameter of the circles used to promote visualization of distance from U, while in [Figure 5] size and colour are varied simultaneously. Actually shades of grey are substituted for colour due to the printing problem.

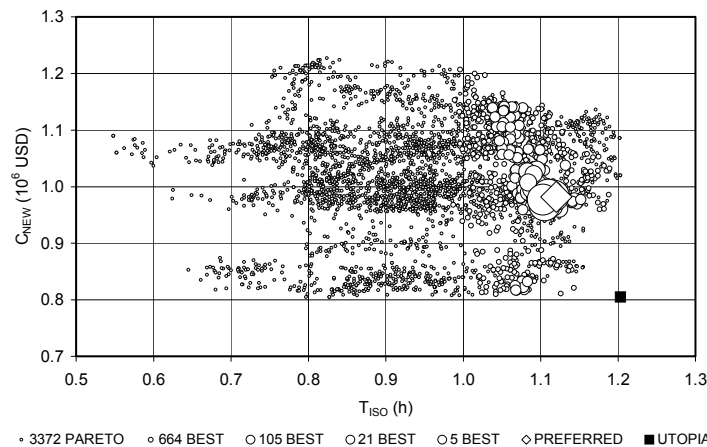


Figure 4. Stratified Pareto-optimal hyper-surface with variable size

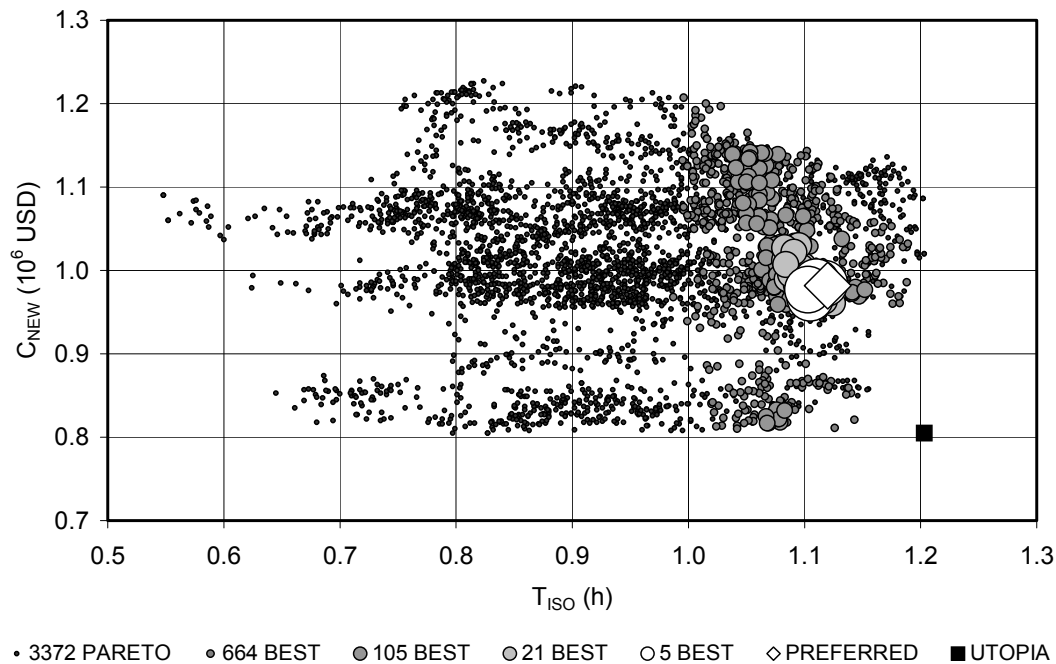


Figure 5. Stratified Pareto-optimal hyper-surface with variable size and colour

7. Conclusions

(1) The hyper-surface of multi-attribute designs may be visualized through stratified symbols. (2) Number of strata should be about five. (3) Graphical sign diameter is best varied according to the modified Renard series. (4) Simultaneous use of size and colour promotes better visualization.

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