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MULTIPLE ATTRIBUTE DECISION MAKING TOWARDS MOST ACCEPTABLE VARIANT SOLUTION

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1. Introduction

Design process includes all phases of design task, from initial idea of possible design solution to final solution using needed analysis and synthesis procedures. Design solution presents generated answer of the design process on given design task inside some technical system. In this process transformation, from idea to principally variant solution, interaction between technical system characteristics and process of solution generating is present. Technical system is defined with exterior and interior properties, which are determining the complexity of the influence on the design solution [Hubka 1988, Duhovnik 2000]. Interior properties are determining the domain of principled variant solutions, which are managed by designer. External properties present the domain of definition of all design solutions and they are in interaction with interior properties. Design process is directed toward connection between functional product's structure and its shape. The basis for interact connection between function and shape of the product make demands and desires, which are described by words and graphically. Described requirements are defined in the rule as a function and/or functionality. Graphical requirements are defined as a shaped model or as known technical system. Functional demands are accepted and defined as axioms in axiomatic design. Therefore, influence parameters of the design process are defined by axioms. Design parameters are defined as design properties. These design parameters have the influence on the design effectiveness and product development through f.e. minimising of all parts of design solutions, modul development and modularity principle use, using of standard components, design of the product has to be technological, simple assembly and disassembly etc. Axiomatic design is presented by three domains, which take completely the part in the flow of the design process. Design process in axiomatic design is usually presented by transformation of customer requirements (CRs) to functionality domain requirements (FRs), domain of physical requirements (DPs) and process variables domain (PVs). Design process as interaction between defined domains is given by matrix equation $\{\mathbf{FR}\} = [\mathbf{A}]\{\mathbf{DP}\}$ according to [Suh 1990]. In this paper details of a new conceptual design model are given.

2. New approach to conceptual phases of design

Model of conceptual design presented in this work is based on the set theory, mathematical logics and mathematical laws. Let the design process as set U define (universum, universal set), and its elements as subsets i.e. as working steps of design process (u_1, …, u_n). In that case design process can be defined as:

$$U = \{u_1, \ldots, u_n\} \tag{1}$$

The conceptual phase is the subset of the set U. If the product is considered as the technical system, the design task determinate an interaction of the working steps into design process during the generating of the design solution as the technical system. Conceptual model is defined by lot of domains, where its number depends on the complexity of the design problem. Presented model of conceptual design in this paper analysis a transformation of the design demands, which are determined by new product design as a technical system into set of design requirements for the generating of different variant solutions. Let us to suppose that technical system is composed from physical domain, biological domain, chemical domain, sociological domain and ΔR (Fig. 1). In such a way of technical system domains presentation it is not intention to determinate all domains, only to show its existence and dynamic altering by structure and content. The domain ΔR presents residuum domain and composes all parts of technical system, which are not explicit given as domains, but they may be defined according to different level of design task complexity. The model of conceptual phases presents the transformation of customer requirements domain into design requirements.

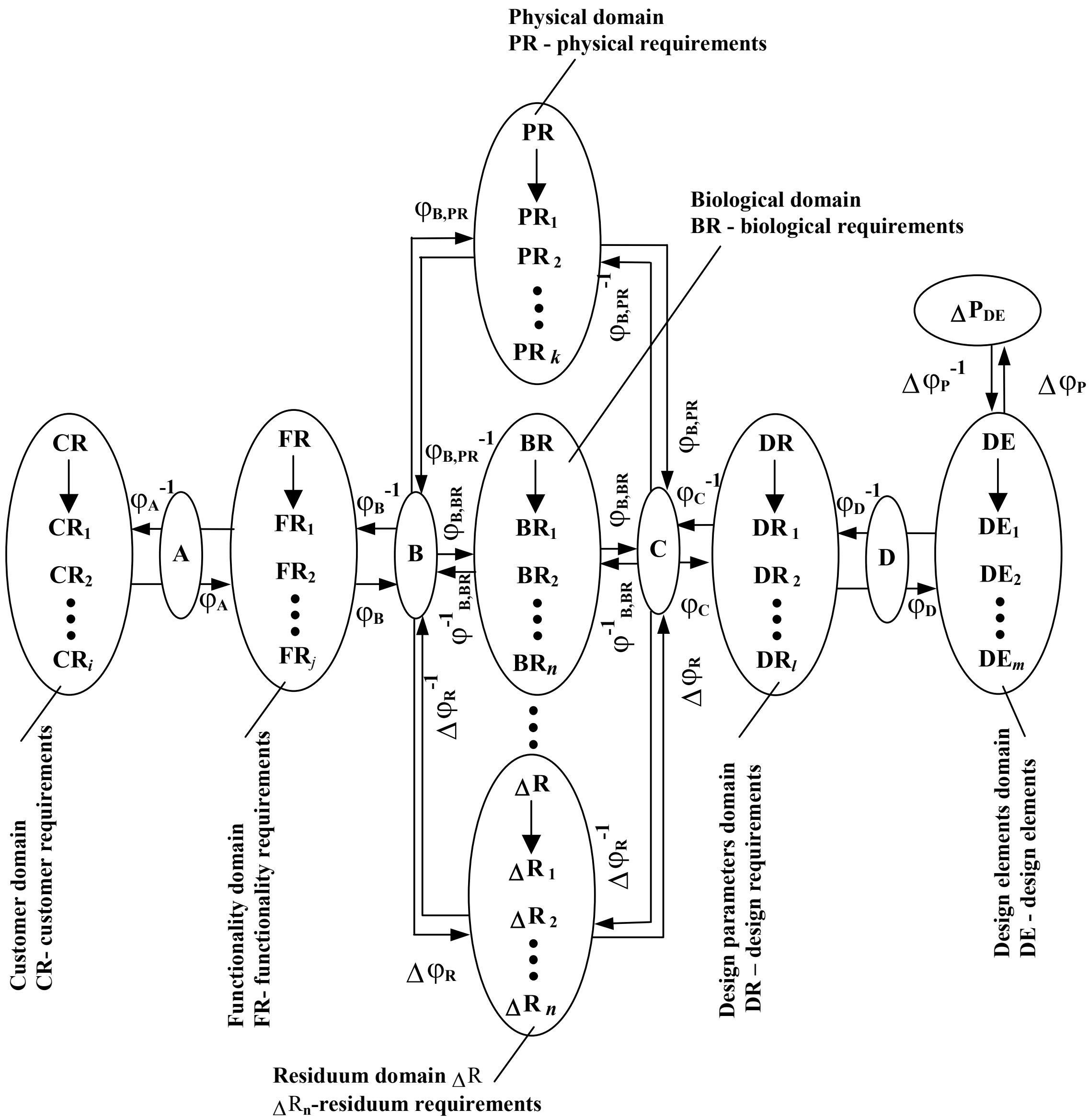

Figure 1. The transformation of the requirements in considered model of conceptual phases

All mathematical relations of the model are more explained and performed in the work [Ivandic 2002]. This phases of conceptual design results with obtained set of principal variant solutions *V*:

$$V = \{V_1,...,V_r\} \tag{2}$$

3. Interaction between variant solutions and evaluation criteria's

Most decision problems involve multiple attributes (criteria). Weights may also be assigned to attributes to represent their relative importance. To assess an alternative on attributes, numerical values or subjective judgements can be used to differentiate one alternative from another. In this section, we use a belief structure to describe subjective assessment information. To this purpose, the model for procedural evaluation the importance of some criteria in conceptual phase of design has been developed. Criteria set (C_k) is conditioned by set of design properties (DP_s). The grade of any criteria gives the answer about utility level of some design property. Preference of one variant solution to another is based on the criteria assessment. The evaluation model includes complete analysis of all alternatives assessing any particular solution according to the same criteria. This overall assessment should enable us to choose the best quality solution with the highest total grade value. The multi-attribute decision analysis demands clear and argued mathematical procedure for criteria selection and grades assignment. Interactive model between variant solutions and criteria set may be presented as in Fig. 2.

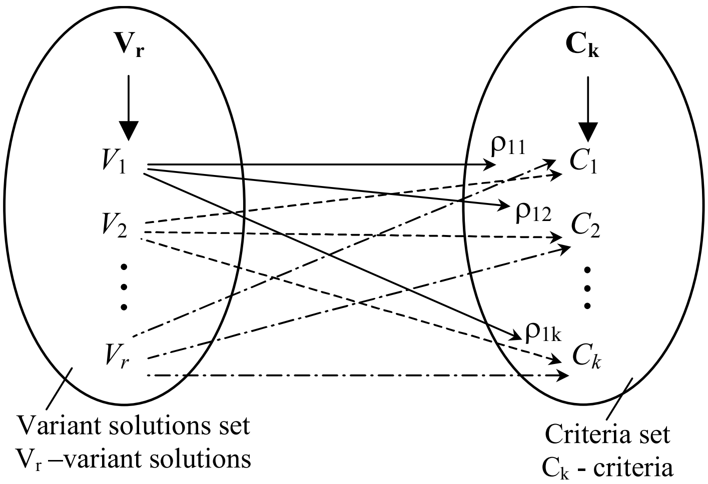

Figure 2. Relations between variant solutions and criteria set

Each principally variant solution $V_1,...,V_r$ has own relations ρ with each criteria $C_1,...,C_k$ from design properties set:

$$\begin{bmatrix} V_1\rho_{11}C_1 & V_1\rho_{12}C_2 & \cdot & V_1\rho_{1k}C_k \\ V_2\rho_{21}C_1 & V_2\rho_{22}C_2 & \cdot & V_2\rho_{2k}C_k \\ \cdot & \cdot & \cdot & \cdot \\ V_r\rho_{r1}C_1 & V_r\rho_{r2}C_2 & \cdot & V_r\rho_{rk}C_k \end{bmatrix} \tag{3}$$

where relation matrix is given by:

$$[R] = \begin{Bmatrix} \rho_{11} & \rho_{12} & \cdot & \rho_{1k} \\ \rho_{21} & \rho_{22} & \cdot & \rho_{2k} \\ \cdot & \cdot & \cdot & \cdot \\ \rho_{r1} & \rho_{r2} & \cdot & \rho_{rk} \end{Bmatrix} \tag{4}$$

The mapping $\varphi_{\rho_{rk}} \to \varphi_{rk}\left(\rho_{rk}\right)$ enables us to write equation (3) in a matrix form:

$$\mathbf{C} = \mathbf{V}^{-1}\,\mathbf{R} \tag{5}$$

what is proven in [Blanuša 1989, Kurepa 1975, Veljan 2001].

4. Grades scale and preferences

The scale of grades and preferences is defined as a set of qualitative and quantitative attributes. An assignment of grades and preferences to attributes leads to assessment of goodness of any variant solution. Quantitative attributes may be measured by using exact numbers, while qualitative attributes are those that may not be readily assessed by using exact numbers in the first instance due to their subjective nature. The preference of any criteria is derived from its importance for variant solution. There are more than one evaluation grades scales and according to [Saaty 1996] 26 different scales are tested and compared. Generally, the scale of quantitative grades may be given as a set:

$$S = \{s_1, s_2, s_3, ..., s_p\} \tag{6}$$

where particular numerical value from the set S for one criteria and one variant solution means the grade of its relation. The scale of evaluation grades may be defined as a set of qualitative attributes S' also:

$$S' = \{s'_1, s'_2, s'_3, ..., s'_q\} \tag{7}$$

The mapping of quantitative values from the set S into set of qualitative values S' is a trivial proof of set elements equality if the next condition is fulfilled:

$$s_{11} \equiv s'_{11}, s_{21} \equiv s'_{21}, ..., s_{p1} \equiv s'_{q1} \tag{8}$$

The matrix of relations **R** has the components as the relations between criteria and variant solutions. Any grade component (H_{ij}) is conditioned by elements of S i S'($H_{ij} \subset S \wedge H_{ij} \subset S'$), where $i = 1,...,r; j = 1,...,k$. All grades composes the $r \times k$ matrix of grades $\left[\mathbf{Q_{rk}}\left(H_{nrk}\right)\right]$ or $\mathbf{Q_{rk}}^{\mathbf{N}}\left[H_{nrk}(\mathbf{f})\right]$. Key role in this evaluation grades model plays the procedure of assignment the grades to attributes, what is usually done by one or more evaluators. Evaluator (E) is in the rule widely educated expert from the multidiscipline team, who is deeply involved in the development of the evaluated product. Each evaluator from the set of n evaluators ($E_1, E_2, ..., E_n$) should assign its grade to each particular criteria of each variant solution. In that case, we have the set of evaluators and the set of grades, whit relations between. In this work, two decision models for grade assignment and preferences definition are compared, according to hierarchic organisation structure. These are centralised decision making model and antagonistic decision making model.

4.1 Evaluation of attributes by centralised decision model

Evaluators belong to the team with central organised structure, which is composed from two levels:

- presentation level, which is presented by the leader of evaluators team and
- level of grade analysing, which is presented by evaluator's analysts for grade assignment.

The presentation level has the task to unite the common grade of hierarchic structure of evaluators for each evaluation criteria related to some known solution. The level of grade analysing presents substantial character of the model, i.e. all of that what analysts have to know about evaluation problem. The relation between grades and evaluators has been performed as a surjection relation, not the bijection. It can be assumed a set *n* of grades $H_{1rk}, H_{2rk}, ..., H_{nrk}$ for *k*-th criteria of *r*-th variant solution assigned by *n* evaluators and *n* real numbers as $p_1, p_2, ..., p_n$, which mean weight factors of some attribute [Ivandic 2002].Real numbers $f_1, f_2, ..., f_n$ are normalised weight factors of criteria grade for particular variant solutions. Grades matrix for *r*-variants and *k*-criteria created from *n* evaluators and *n* $(f_1, f_2, ..., f_n)$ normalised weight factors is determined as:

$$Q_{rk}{}^{N}\left(H_{nrk}\left(\mathbf{f}\right)\right)=\begin{bmatrix} \sum_{i=1}^{n} f_i \cdot H_{i11} & \sum_{i=1}^{n} f_i \cdot H_{i12} & .. & \sum_{i=1}^{n} f_i \cdot H_{i1k} \\ \sum_{i=1}^{n} f_i \cdot H_{i21} & \sum_{i=1}^{n} f_i \cdot H_{i22} & .. & \sum_{i=1}^{n} f_i \cdot H_{i2k} \\ . & . & . & \\ . & . & . & \\ \sum_{i=1}^{n} f_i \cdot H_{ir1} & \sum_{i=1}^{n} f_i \cdot H_{ir2} & .. & \sum_{i=1}^{n} f_i \cdot H_{irk} \end{bmatrix} = \mathbf{Q_{rk}}^{\mathbf{N}}\left[H_{nrk}\left(\mathbf{f}\right)\right]{}_{r}^{k} \qquad (9)$$

Equation (9) presents mathematical formalisms in matrix form needed to describe evaluating of *k* criteria for *r* variant solutions by *n* evaluators.

4.2 Evaluation of attributes by antagonistic decision model

When evaluators belong to the team, which applies antagonistic model of decision making, every member of the team determinate the grades absolutely independent for itself. In this way one incoherent and unconnected team with opposite means is created. Characteristic of such an antagonistic model is the freedom of decision making without constraints and mid-phase control. The reasons for incoherency might be different interests to the assessment of evaluated variant solutions, but also cultural, educational, personal etc. reasons inside the evaluator's team. Each evaluator defines two numerical values for each criteria grade independently:

- the first numerical value presents so-called minimal level (reservation) of grades factor, pointed as $R_j\left[\mathbf{Q_{rk}}^{\mathbf{N}}\left[H_{nrk}\left(\mathbf{f}\right)\right]\right]$, where R_j is numerical value $\mathbf{Q_{rk}}^{\mathbf{N}}\left[H_{nrk}\left(\mathbf{f}\right)\right]$ for K_k criteria on the reservation level
- the second numerical value, which presents maximal level (aspiration) of grades factor. This value has to be converged as one desired value, pointed as $A_j\left[\mathbf{Q_{rk}}^{\mathbf{N}}\left[H_{nrk}\left(\mathbf{f}\right)\right]\right]$, where A_j is value $\mathbf{Q_{rk}}^{\mathbf{N}}\left[H_{nrk}\left(\mathbf{f}\right)\right]$ for K_k criteria on the aspiration level. The matrix of grades on the reservation level is given by:

$$\mathbf{R_j}=\begin{bmatrix} R_{11}\left(H_{n11}\left(\mathbf{f}\right)\right) & R_{12}\left(H_{n12}\left(\mathbf{f}\right)\right) & . & R_{1k}\left(H_{n1k}\left(\mathbf{f}\right)\right) \\ R_{21}\left(H_{n21}\left(\mathbf{f}\right)\right) & R_{22}\left(H_{n22}\left(\mathbf{f}\right)\right) & . & R_{2k}\left(H_{n2k}\left(\mathbf{f}\right)\right) \\ . & . & & . \\ & & . & \\ . & . & & . \\ R_{n1}\left(H_{nr1}\left(\mathbf{f}\right)\right) & R_{n2}\left(H_{nr2}\left(\mathbf{f}\right)\right) & . & R_{nk}\left(H_{nrk}\left(\mathbf{f}\right)\right) \end{bmatrix}, j=1,...k \qquad (10)$$

The matrix of grades on the aspiration level is given by:

$$\mathbf{A_j} = \begin{bmatrix} A_{11}\left(H_{n11}(\mathbf{f})\right) & A_{12}\left(H_{n12}(\mathbf{f})\right) & . & A_{1k}\left(H_{n1k}(\mathbf{f})\right) \\ A_{21}\left(H_{n21}(\mathbf{f})\right) & A_{22}\left(H_{n22}(\mathbf{f})\right) & . & A_{2k}\left(H_{n2k}(\mathbf{f})\right) \\ . & . & & . \\ & & . & \\ . & . & & . \\ A_{n1}\left(H_{nr1}(\mathbf{f})\right) & A_{n2}\left(H_{nr2}(\mathbf{f})\right) & . & A_{nk}\left(H_{nrk}(\mathbf{f})\right) \end{bmatrix}, \; j = 1,...k \qquad (11)$$

Derived grade value presents the arithmetical mean value:

$$\mathbf{Q_{rk}}^{\mathbf{N}} = \frac{\sum_{j=1}^{n} R_{ij}\left(\mathbf{Q_{rk}}^{\mathbf{N}}\left[H_{nrk}(\mathbf{f})\right]\right) + \sum_{j=1}^{n} A_{ij}\left(\mathbf{Q_{rk}}^{\mathbf{N}}\left[H_{nrk}(\mathbf{f})\right]\right)}{i}, \; i = 1,...n; \;\; j = 1,...,k \qquad (12)$$

(where are *j* – the number of criteria ($j \le k$) and *i* – the evaluators number $i = 1,...,n$).
This solution is quasi-satisfied solution, because it presents the compromise in variant solutions evaluation between minimal and maximal grades values.

5. Conclusions

New approach to conceptual design and decision making model presented here includes all complexity of product development as the technical system. It is based on mathematical formalisms of the goodness of some variant solution toward given criteria. Proposed model assures clearly defined procedure for grades assignment, with the aim to rank all variant solutions. Highest value of the grade obtained by evaluation procedure means the best solution. Such approach in original form unites conceptual phases of design and evaluation of alternative solutions, what makes the time of product development shorter. Working examples of applying these grading models may be found in the reference [Ivandic 2002].

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