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LINKING ELASTIC DEFORMATIONS AND FABRICATION DEVIATIONS IN COMPUTER-AIDED-ANALYSIS

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Abstract

The following paper presents a conceptual study of linking elastic deformations and fabrication deviations. The essential base is represented by the description of a technical surface with mathematical tools. Such surface functions will be set up for both elastic influences and fabrication contingent deviations. Through the principle of superposition a "total function" is produced, from which the four statistical moments can be calculated. Those are integrated into a commercial tolerance analysis system, which simulates the assembly process of the whole assembly.

Keywords: tolerance modelling, variational modelling, functional modelling, simulation

1 Introduction

In many application areas of technology the consideration of both elastic deformations and fabrication deviations is very important. Especially while assembling single components to a whole device, both phenomena appear coupled. On the one hand the component surfaces are afflicted with fabrication deviations, on the other hand deformations arise caused by the assembly situation (for example screwing, riveting…). These deformations which are not demonstrable around an unencumbered condition influence the assembly result significantly. In order to be able to represent this situation close to reality in the computer-aided analysis, it is obvious that only a hybrid simulation can fulfil this circumstance.

In this contribution, a concept is introduced how a reasonable coupling of elastic deformations and fabrication deviations can be performed. Based on a simple computer-internal example both the computer-aided-implementation and an exemplary simulation are presented.

2 Concept

The mathematical description of a technical surface forms the fundament of the draft. Based on waviness functions these surfaces can be generally described as follows [1]:

$$y(x) = \sum_k b_k \cdot \sin\left(\frac{2p}{l_k} \cdot x + j_k\right) \tag{1}$$

This formulation possesses validity for the two-dimensional application and acts on the assumption that waviness repetitions appear periodically. A line on the surface described in that way must be set up for both the fabrication deviations and the elastic deformations. The base therefore is identical in both cases, because data is required, which results from the detection of surface points.

The won information must now be prepared in a suitable manner so that a sensible superposition of the phenomena fabrication deviations and elastic deformations is given. There exist three basic possibilities that are clarified more closely later in this contribution [2].

Out of the developed superposition function four statistical moments can be calculated [3], whereby the last three of them are of a special importance. These can be integrated into a commercial tolerance analysis system, which is requisitioned to realize the hybrid simulation. Thereby a statistical analysis of the assembled components which is close to reality is enabled. The superposition takes place respectively for every joint surface whereby the remaining body is to be looked at as stiff. Figure 1 clarifies the concept.

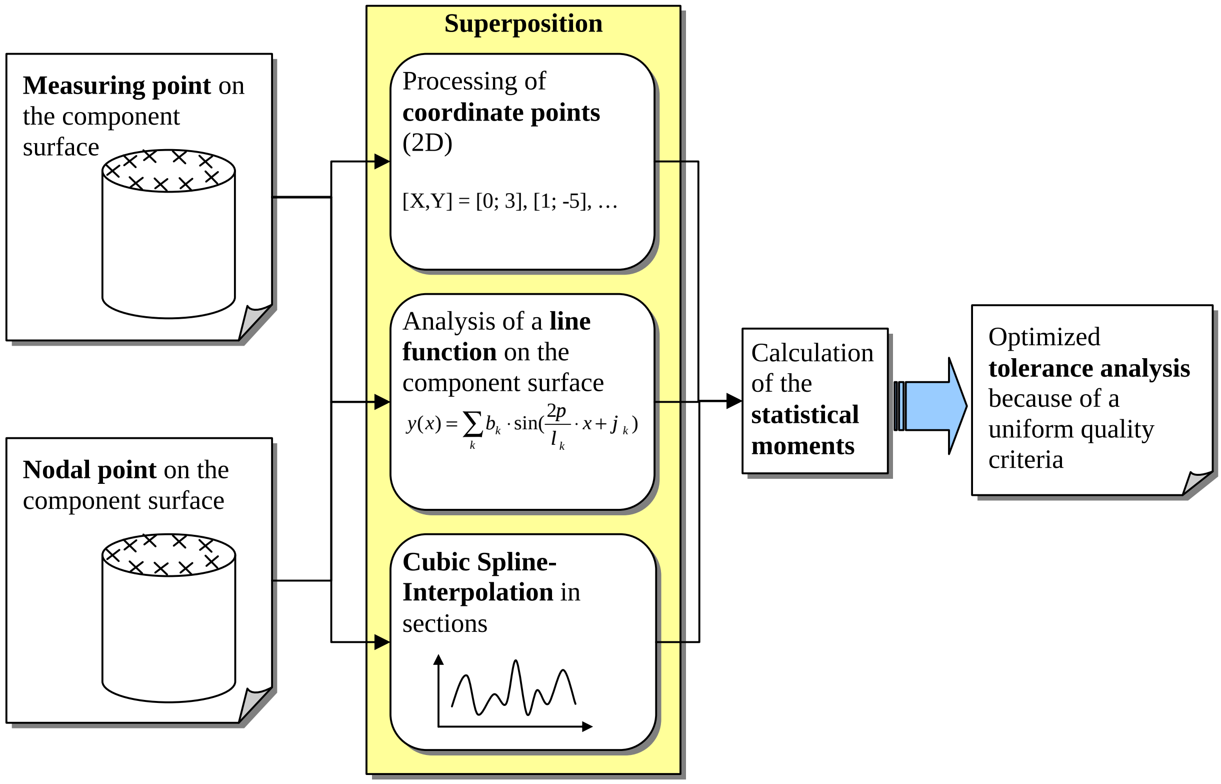
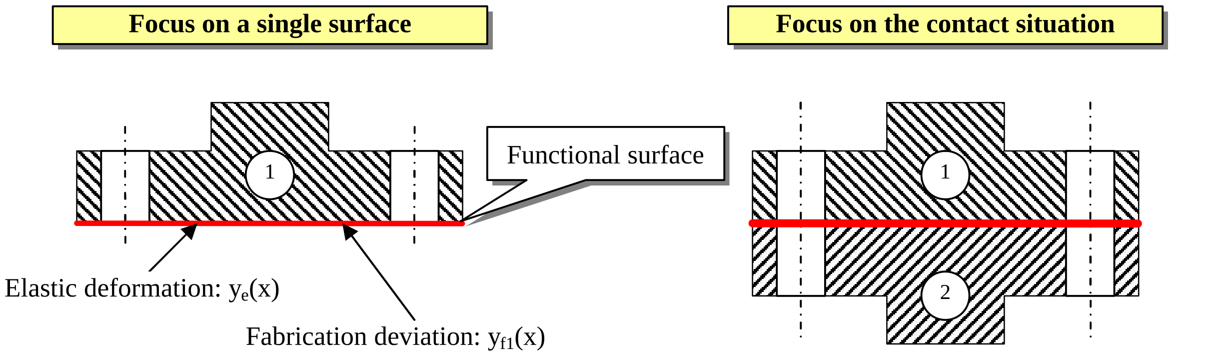


Figure 1. Basic concept

Usually fabrication deviations can be collected through the use of a coordinate measuring machine. Thereby a profile of a line on the component surface can be represented. Out of it the appendant function is determined.

Elastic deformations are calculated computer supported by using the finite element method. Here the mathematical function is also created using a surfaces profile, on which all displacements, which result throughout the assembly situation, are applied.

The coupling of both attained functions represents figure 2. First each involved contact area is observed alone. Here the actual superposition of both phenomena takes place, what leads to the first two assumptions, that are necessary for a successful coupling: first the statistical parameters skewness and kurtosis must be calculable (besides the nominal position and the standard deviation) out of the mathematical description of the line on the component surface. Secondly the data points must be represented in a manner that they can be superposed mathematically. In the subsequent assembly process, in which now both contact areas play a role, assumption number three must be fulfilled: after the superposition it must be possible to integrate a phase difference, in order to filter out the assembly variant with the best result (minimum of the standard deviation).



$$f_1(x) = y_{f1}(x) + y_e(x)$$

$$f_2(x) = y_{f2}(x) - y_e(x)$$

$$f_1(x) + f_2(x + J)$$

Figure 2. Fundamental procedure during the superposition

In order to realize a mathematical conjunction of fabrication deviations and elastic deformations, there exist three possibilities in general.

The processing of the point data is considered as the simplest possibility. By implementing an addition of the space coordinates and subsequently the ascertainment of the statistical parameters, a superposition can take place. But the integration of a phase difference is impeded, because one point can only be connected with the respectively next. Due to this no phase difference lying between more data points can be simulated. Consequently the pure point data has to be further prepared by interpolating a function between the sampling points. A cubical spline-interpolation can help along here, but it is only defined segmental [4] and therefore leads to problems in the integration of a phase difference again.

Ideally the description of a line on the component's surface exists now as a mathematical function. The mathematical superposition can be performed and out of it the resulting statistical moments can be calculated.

After the superposition of the waviness functions, the view on the complete assembly follows (contact situation of two joining surfaces). The parameters "standard deviation", "skewness" and "kurtosis", that were previously calculated, can now be inserted directly into a tolerance analysis system (figure 3).

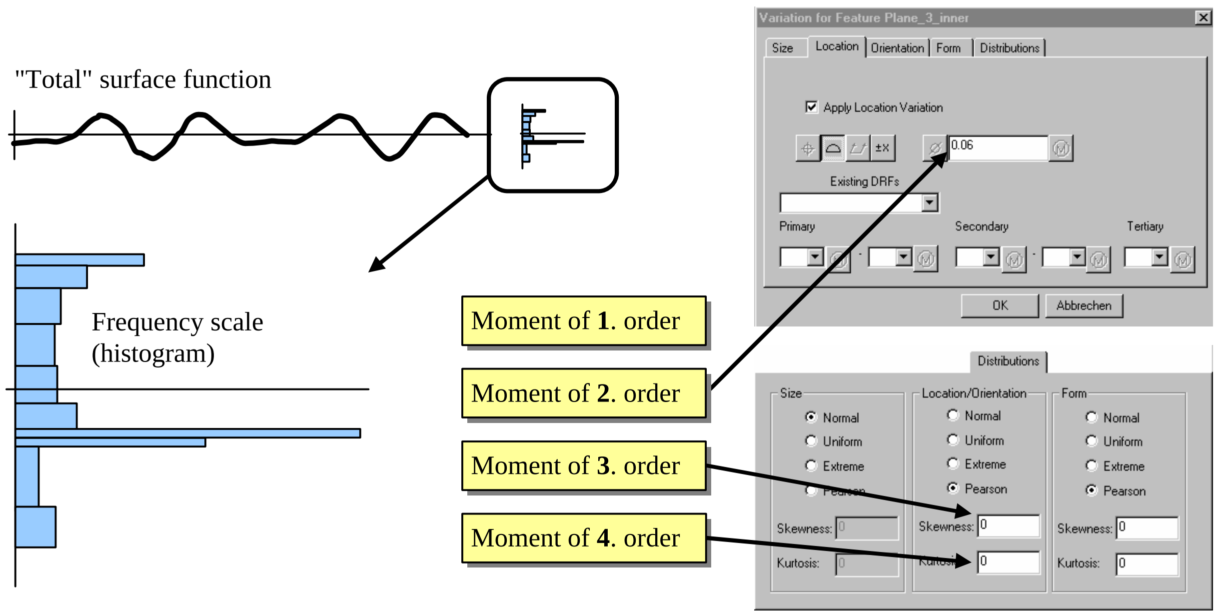


Figure 3. Integration of the statistical moments in a tolerance analysis system

Now the effect of a phase difference can be analyzed. In simple, for example rotation symmetrical components that are connected by screws, a new assembly variation can be produced by recalculating the relevant statistical parameters. The two joining surfaces therefore lie above each other in a new position. Certainly it has to be ensured that a twisting of the components is only possible from hole to hole, because otherwise in reality no more screwing would be possible.

The basic sequence of the tolerance analysis remains untouched in spite of the additional data (statistical parameters). First the geometry model of the component is transferred from a CAD system into the tolerance analysis module. Here special CAD translators enable the data exchange. Now all contact areas that are involved in the assembly process must be assigned. Each contact area is afflicted with standard tolerance indication. Of course these indications can be modified afterwards in favour of the user's wishes. After that the assembly sequence must be defined by assigning the areas that are lying on each other. In the last step before the start of the simulation, the user can define measurements, whose results are of a special importance to him. Based on tolerance and assembly influences for example the detection of the variation of an angle or an interval are possible measurement types. After terminating the simulation, the results can be analysed and the model will be optimized.

3 Realization of an demonstrating example

For demonstration purposes, a relatively simple assembly is used. It consists of ten circle rings, which are mounted and screwed on each other. Each ring has 16 drill holes to integrate elastic influences that result from the screw connection (figure 4). The top surface is afflicted with a parallelism tolerance, referencing to the bottom surface. The tolerance value, represented by the standard deviation, is calculated and used for each assembly variant.

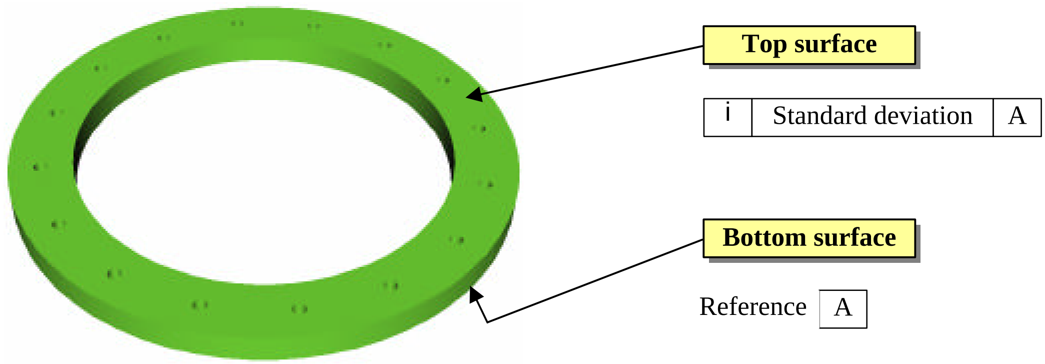


Figure 4. Ring-shaped demonstrator with pertinent tolerancing

The fabrication deviations that can emerge through mechanical oscillation during the manufacturing process are described as a so called "three waviness". The following function represents this fact:

$$y_f(x) = 0{,}015 \cdot \sin\left(\frac{0{,}428}{50}x\right) \tag{2}$$

The wave length here is 733.04 mm. If 2π is divided by this value, you exactly receive the result that the fraction delivers in the above-mentioned function. It's only a simplified representation. The amplitude of 0.015 covers the ±3σ area.

Based on the 16 drill holes a 16-waviness appears in the assembled condition, which can be described approximately by the following function:

$$y_e(x) = 0{,}01 \cdot \sin\left(\frac{2{,}286}{50}x + 1{,}5\right) \tag{3}$$

The amplitude of the function is calculated by means of a finite element analysis. The maximal node displacement is about 0.02 mm. In the denominator of the fraction, the wave length appears, that is 137.44 mm in a circle with a diameter of 700 mm and an angle each with 22.5°. As a phase difference the value 1.5 is selected, which only has a correcting character, because the maximal eruption should exactly lie on the ordinate.

Both functions of the fabrication deviations and elastic deformations are added now:

$$y_1(x) = y_f(x) + y_e(x) = 0{,}015 \cdot \sin\left(\frac{0{,}428}{50}x\right) + 0{,}01 \cdot \sin\left(\frac{2{,}286}{50}x + 1{,}5\right) \tag{4}$$

The behaviour of a functional surface is known. Now the cooperation of two surfaces in the assembled condition is considered. Additionally one of the surfaces will be fitted with a phases difference of 22.5°. This complies with the angle that lies between two of the 16 drill holes. The superposition-functions can be visualised in a chart. The graphs of the phase angles 0°, 22.5° and 45° are represented in figure 5:

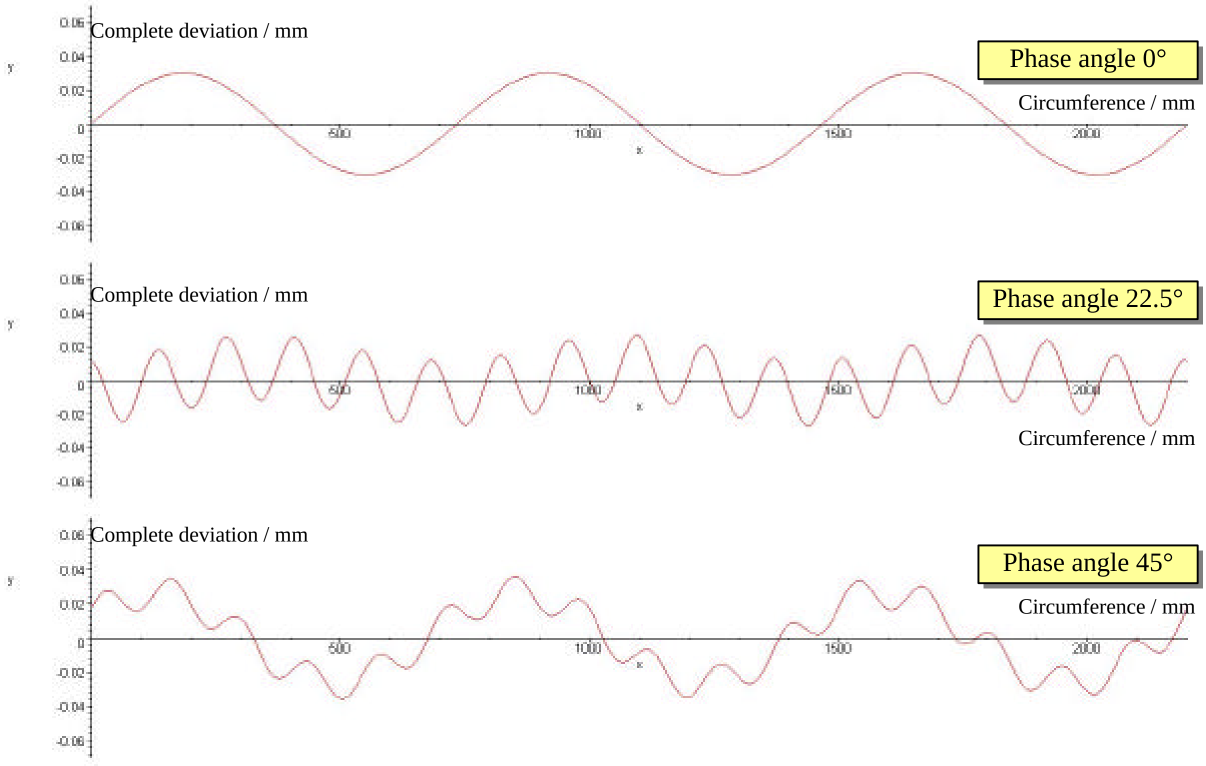


Figure 5. Superposition curves for some selected assembly variants

Out of the superposition functions the statistical moments can now be calculated. In order to be able to determine the required parameters, sample values of the above-mentioned functions are necessary. The exact value for the statistical moments exists if all points of the function graphs are observed. Because this amount would be too large for calculation purposes, the sample value is taken only after a certain increment step and is assigned to the sample. While in large increment steps (3-10) the parameters vary too much, the value converges if the increment steps are selected smaller. However the calculating effort climbs considerably when using smaller steps. In this example, the expanse 2.5 was selected. That means that every x+2.5 (beginning with x = 0) the respective function value is read and is registered into a list. The number 2.5 was selected therefore in order to be able to consider on the one hand both even and odd x values and on the other hand to reduce calculating effort to a rational minimum.

Now the calculated statistical moments are integrated into the tolerance analysis system VisVSA™ for every 16 assembly variants. The commercial tolerance analysis module offers the possibility to integrate not-normal distributed data (for example the parameters skewness and kurtosis). After the import process of the geometry model, all assembly steps of the whole assembly have to be explicitly defined again. Afterwards the user can generate own measurement characteristics in order to obtain the desired results.

From the constructive sight above all the parallelism deviation of the whole assembly is of a special interest. The functionality could be programmed in C++ and could be integrated subsequently into VisVSA™ [5].

4 Simulation and study of parameters

The simulation works by using a Pearson distribution. This can be characterized for each assembly variant (phase difference) through the choice of the parameters skewness and kurtosis besides the definition of the nominal geometry and the standard deviation. The four statistical moments were already calculated and therefore they are available for simulation purposes. All contact areas that are located between each pair of rings are occupied with the mentioned value couple as well as the standard deviation is used for the tolerance zone. In each newly started simulation, the parameters are adapted and the results are stored in terms of the process capability values (c_p and c_{pk}) in order to be able to give a quality statement. In addition the standard deviation of the performed measurement is recorded. In VisVSA™ this is the result for the size of the tolerance zone and represents the parallelism deviation of the whole assembly.

Figure 6 shows the results of the Monte-Carlo and High-Low-Median-simulation for a phase angle of 0° (integrated statistical values: standard deviation = 0.02122; skewness = 0.000007599; kurtosis = 1.4986).

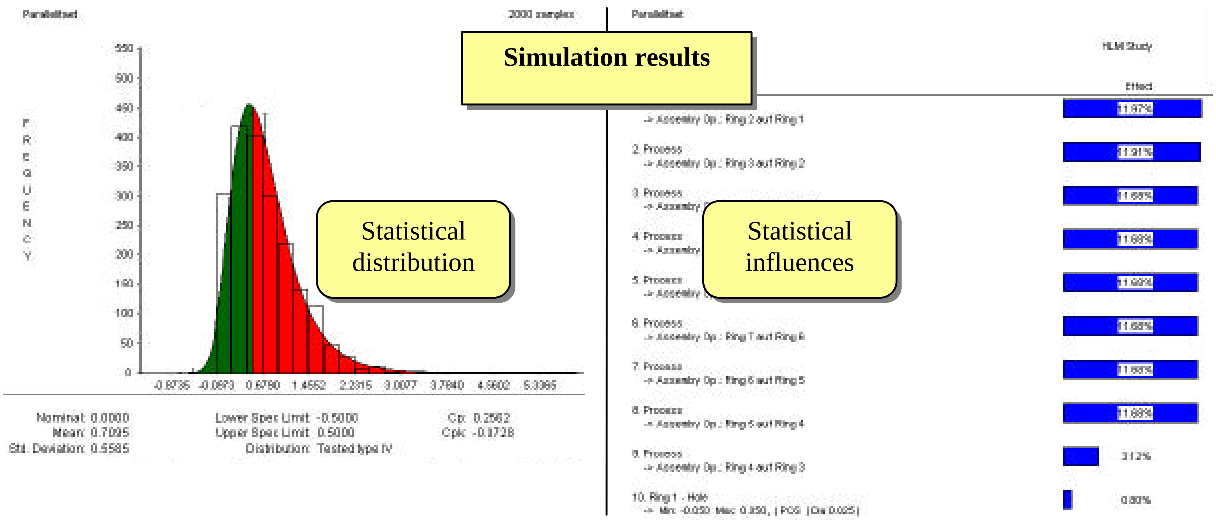


Figure 6. Measurement of parallelism considering statistical influences (phase angle 0°)

Table 1 delivers a comparison of selected values of the results with and without consideration of additional statistical data:

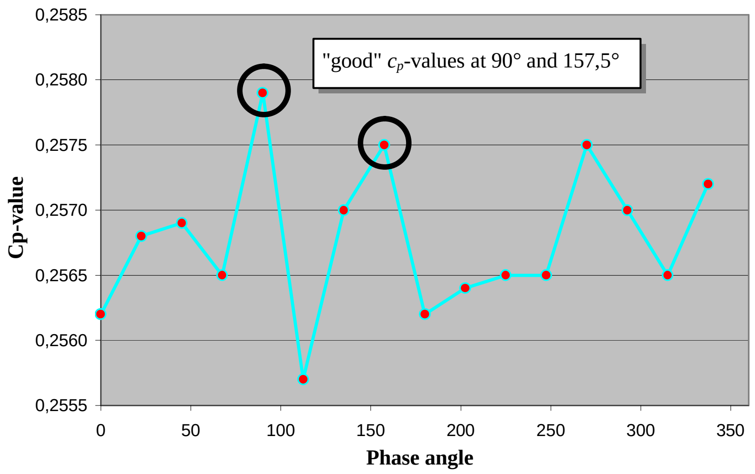
Table 1. Comparison of selected parameters

	Normal distribution	**Pearson distribution**
Expected value	0.7121	0.7095
Standard deviation	0.5573	0.5585
c_p-value	0.2561	0.2562
c_{pk}-value	-0.0739	-0.0728
Out of specification	57.7%	57.4%

The assertion can be posted that a more exact simulation model is created by the influences resulting from the parameters "skewness" and "kurtosis".

Especially in this example it is noticeable that a consideration of elastic deformations coupled with fabrication deviations leads to negligibly better results. The rate of the performed assemblies that lies outside of the specifications decreases about 0.3%. At the same time the process capability value c_p increases from 0.2561 to 0.2562. The parallelism deviation of the whole assembly (standard deviation) is slightly higher during the use of the Pearson distribution. That is to be led back on the fact that the assembly steps can be performed successfully, but additionally the integrated elasticity information and fabrication deviations influence the simulation.

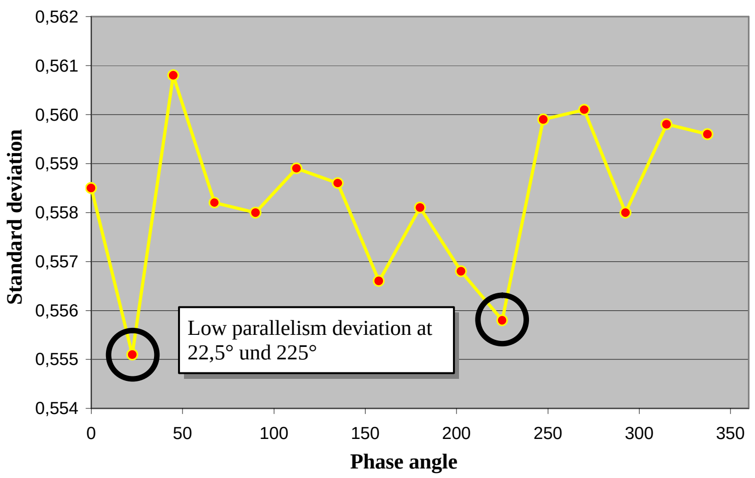
Every possible assembly variant is simulated and the results are stored. The outcoming curve for the process capability is presented in figure 7:



Phase angle	c_p-value
0°	0.2562
22.5°	0.2568
45°	0.2569
67.5°	0.2565
90°	0.2579
112.5°	0.2557
135°	0.2570
157.5°	0.2575
180°	0.2562
202.5°	0.2564
225°	0.2565
247.5°	0.2565
270°	0.2575
292.5°	0.2570
315°	0.2565
337,5°	0.2572

Figure 7. Characteristics of the process capability value in different phase angles

The higher the process capability value is, the better the assembly can be fulfilled. If there is the only aim to optimize the process, this criterion would suffice in order to encounter a quality statement. The assembly result itself is not considered here, but however plays a major role in this example. Additionally on this account the statistical moments have to be calculated for each phase angle. The process of the standard deviation is represented in figure 8:



Phase angle	Standard deviation
0°	0.5585
22.5°	0.5551
45°	0.5608
67.5°	0.5582
90°	0.5580
112.5°	0.5589
135°	0.5586
157.5°	0.5566
180°	0.5581
202.5°	0.5568
225°	0.5558
247.5°	0.5599
270°	0.5601
292.5°	0.5580
315°	0.5598
337.5°	0.5596

Figure 8. Standard deviation for all assembly variants

Certain coherences between the standard deviation (therefore the parallelism deviation of the whole assembly) and the c_p-value are to be recognized, also if there is no perfect identity. The lowest parallelism deviation is to be detected in the 22.5° position whereby the c_p-value is also located in a relative good area. But according to figure 7 the optimum would be at about 90° (highest c_p-value); in the standard deviation chart, this value does not rank in the upper third, but rather is more an average result.

In Table 2 the sequence is represented, in which angle two rings should be assembled, in order to minimize the deviation.

The slightest parallelism deviation is given if the assembly is mounted distorted for 22.5°. Very good assembly results are also given when two rings are mounted in the 225° or 157.5° position.

Table 2. Ranking for choosing an assembly variant

Rank	**Phase**	**Parallelism deviation**	**c_p-value**
1	22.5°	0.5551	0.2568
2	225°	0.5558	0.2565
3	157.5°	0.5566	0.2575
4	202.5°	0.5568	0.2564
5	90°	0.5580	0.2579
6	292.5°	0.5580	0.2570
7	180°	0.5581	0.2562
8	67.5°	0.5582	0.2565
9	0°	0.5585	0.2562
10	135°	0.5586	0.2570
11	112.5°	0.5589	0.2557
12	337.5°	0.5596	0.2572
13	315°	0.5598	0.2565
14	247.5°	0.5599	0.2565
15	270°	0.5601	0.2575
16	45°	0.5608	0.2569

A comparison between the "worst" and the "best" assembly situation shows that an improvement about 1% can be obtained, if certain assembly situations are preferred.

5 Conclusion

Based on the results the meaningful coupling of elastic deformations and fabrication deviations in the computer aided tolerance analysis can be considered as very necessary. Especially when using self-programmed functionalities, the designer gets an expanded tool with which he can reduce the complexity of long tolerance chains to a clear size. One point to be emphasized is that by using the introduced method the model development passes more exact and so the simulation gets more closely to reality.

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