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Monte Carlo Simulation of Multi-Criteria Decisions Under Uncertainty

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Abstract
In today's world of engineering design, two key properties govern decisions, i.e. uncertainty and multiple decision criteria. In the scope of this paper, a method based on Monte Carlo simulation is proposed to evaluate and improve multi-criteria decisions under uncertainty in engineering design. This is based on determining the effects of a decision and characterizing these using probability distributions and probabilities of occurrence. Depending on the value of influences, the resulting probability distribution for the decision criteria will be different. The proposed method based on Monte Carlo simulation allows the user to evaluate and improve the alternatives of a decision. Each alternative is characterized as a set of influence/value combinations. It is shown how the method has been implemented at a manufacturer of semiconductor equipment to support decisions in modular product development.

Keywords: Decision-support, uncertainty, robust design, Monte Carlo simulation.

Introduction
In this section, the fundamental characteristics of decisions in engineering design are shown up. The require a paradigm shift from optimization to satisficing. The requirements for a decision support method to operate within this environment are elaborated on.

Characteristics of Decisions in Engineering Design
The number of stakeholders of decisions in engineering design continues to increase. No longer are decisions made in a single location. Engineers increasingly work in distributed teams across multiple R&D sites in different countries and continents [1]. With the advent of mass customization [2], companies are also striving to suit to the tastes of individual customers. Finally, companies are increasingly making use of product families and platforms [3] in order to make use of economies of scale across multiple products. The result of all these tendencies is an increasing number of criteria to be considered in making decisions in engineering design.
The above tendencies also increase uncertainty. As a result of the increasing number of different stakeholders, customers, and products, the number of potential changes that may occur in the life cycle of a product is increasing. The environment that a product is designed into is more and more uncertain.
In the above-described, optimization is only feasible for decisions of very limited scope. The assumption of all optimization methods is that the underlying optimization model is

sufficiently close to reality to serve as a basis for decision making. As known from nonlinear control theory [4], a complex system may jump between multiple modes of operation. In optimization, however, you "optimize" for a single state of operation without considering that this represents no accurate model of reality. Not only are the results of optimization false, they are even misleading and dangerous. The more a system is optimized, the more it becomes specialized towards a particular mode of operation and fragile in all others. Also, optimization requires all alternatives being measured with one common utility function. This requirement cannot be fulfilled in a multi-faceted environment. That is why we need to move from optimization to satisficing (Figure 1) as recognized by Simon. "We cannot within practicable computational limits generate all the admissible alternatives and compare their respective merits. Nor can we recognize the best alternative, even if we are fortunate enough to generate it early, until we have seen all of them. We satisfice by looking for alternatives in such a way that we can generally find an acceptable one after only moderate search." [5]

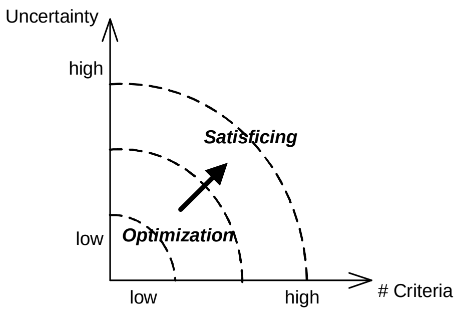

Figure 1. Paradigm shift from optimization to satisficing.

Consequently, there is a need for a method allowing the decision maker to find such an acceptable alternative.

Requirements for a Decision Support Method

A decision support method for satisficing should fulfill the following requirement.

- *Allow the assessment of alternatives based on multiple decision criteria*: The decision support method should allow assessing different alternatives within a decision based on multiple identified decision criteria.
- *Help improve alternatives according to the decision criteria*: Unlike optimization, satisficing is more than just choosing one solution among a set of alternatives. It is about considering multiple alternatives and improving one or several of them until all decision criteria reach satisfactory levels. That is why a decision support method should not only allow the comparison of different alternatives by how they perform on various criteria. It should also help improve alternatives.

In fulfilling the above two functions, the method should also have the following characteristics.

- *Be quantitative*: In our view mathematics, is a clear – possibly the only clear – language to transmit and process information. That is why a method to handle information in decision making should preferably be mathematic and therefore quantitative.
- *Incorporate uncertainty and risk*: Hazelrigg [6] defines uncertainty as the absence of precise knowledge about the effects of decisions. Risk is insecurity about the outcome of the decision criteria resulting from uncertainty. Uncertainty and risk are crucial characteristics of decisions in engineering design that need to be acknowledged.
- *Be industrially applicable*: The objective in conceiving a method for decision support in engineering design is to improve decisions in engineering design in industry. The method should be readily applicable and thus be easy to understand and realizable with

acceptable effort. Besides, software support should be provided that neither requires additional investments nor training.

Background

Four concepts currently used to support decisions in engineering design are described next. The advantages and disadvantages of these methods are shown up. It is described how these methods can be developed further in view of the requirements set out in the previous section.

Decision Tables

Decision tables are among the simplest concepts for decision analysis. In a decision table, the various alternatives of a decision are listed vertically and the decision criteria horizontally. Evaluations of how each individual alternative score on each of the criteria are placed into the resulting matrix. Owing to this simple matrix structure, a decision table can be easily augmented with additional alternatives or criteria. Note that the evaluation of the fundamental concepts in Figure 4 is actually an example of a decision table.

A decision table allows for a quantitative assessment of alternatives (Figure 4). It does, however, not allow improving alternatives, as the influencing factors to the decision criteria are not shown. Since a single number is given to an alternative for each criterion, uncertainty and risk cannot be incorporated into decision tables. Finally, decision tables are industrially applicable as they can be easily implemented using simple spreadsheet tools.

Decision Trees

Unlike decision tables, decision trees not only allow for multiple alternatives, but also incorporate branches for different uncertain outcomes. As shown in Figure 2, Alternative 1 has a 0.4 chance of leading to outcome 10 and a 0.6 chance of leading to 5.

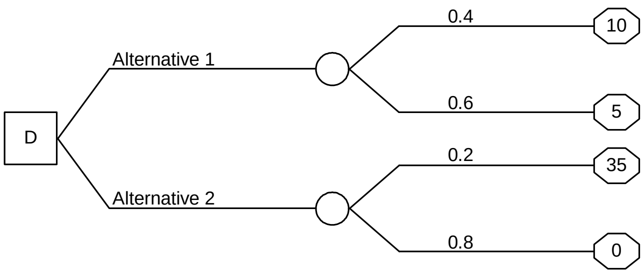

Figure 2. Example of decision tree.

Uncertainty can thus be accounted for in decision (Figure 4). Risk may also be considered by augmenting the tree with utility theory and the concept of the certain equivalent [7]. Decision trees are, however, targeted towards a single criterion and do not allow the assessment of alternatives based on multiple criteria. The most significant drawback of decision trees is industrial applicability. Although there are commercial software packages available for decision trees (e.g., TreeAge or PrecisionTree), decision trees grow exponentially and can only be used for decisions of modest scope.

Influence Diagrams

Influence diagrams, which are very similar to relevance diagrams, belief nets, and Bayesian networks, are singly connected, acyclic, directed graphs with decision, chance, and value nodes and conditioning and informational arrows between them [8]. In Figure 3, an example of an influence diagram with decision node D, chance node C, and value node V is shown. An informational arrow describing available information is between C and D. Conditioning arrows denoting probabilistic relevance are between D and C, C and V, as well as D and V.

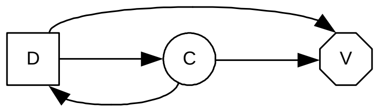

Figure 3. Example of influence diagram.

Influence diagrams primarily show the relationships that affect the decision criteria. They are therefore very helpful for improving alternatives, but less useful in assessing different alternatives based on multiple criteria (Figure 4). Influence diagrams allow the incorporation of uncertainty and risk. Commercial software support for influence diagrams is available (e.g., Analytica or DecisionPro). Besides, influence diagrams are very suitable for an industrial context, because they are graphic and simple to grasp. Conceptually, influence diagrams are quantitative, although in practice they are mostly qualitative.

Simulation

All previously discussed concepts are for analysis, i.e. the decision criteria are evaluated for a one-shot run of the decision. The fundamental difference in simulation is that you execute multiple runs to see how the system behaves in different situations. Consequently, simulation is closer to reality than the previously mentioned concepts [9].

Using simulation, different alternatives of a decision can be assessed for multiple criteria (Figure 4). These alternatives can then be improved, because the relationships influencing the decision criteria are laid down in the simulation model. Virtually all simulation models are quantitative. Uncertainty and risk can be incorporated into simulation by executing multiple runs. Unlike the previously discussed concepts, simulation does not imply a concrete method. Simulation needs to be particularized for the actual decision at hand.

	Assessment of alternatives	Improve alternatives	Quantitative	Uncertainty and risk	Industrially applicable
Decision tables	●	○	●	○	●
Decision trees	○	●	●	●	○
Influence diagrams	○	●	●	●	●
Simulation	●	●	●	●	○

Figure 4. Evaluation of concepts.

Monte Carlo Simulation of Multi-Criteria Decision

Each of the previously introduced concepts has its specific advantages. The method described in the following combines the advantages of influence diagrams and simulation. Influence diagrams are used to easily gather and process information on the decision system while simulation is used to quantitatively assess and improve alternatives as well as managing uncertainty and risk. The system architecture of the method and definitions of the terms used are given first. Then, the functioning of the method in simulating decisions is discussed.

System Architecture

The system architecture of the proposed method for Monte Carlo simulation of multi-criteria decisions is shown in Figure 5. As in Fernández et al. [10], it is distiguished between two primary types of decisions, i.e. selection and compromise. A selection consists of choosing among a number of alternatives. A compromise consists of improving an alternative through its modification. Selection and compromise very often appear together. The term decision is therefore defined the following manner.

- *A decision is the selection among multiple alternatives and/or the improvement of alternatives through compromise according to one or several decision criteria.*

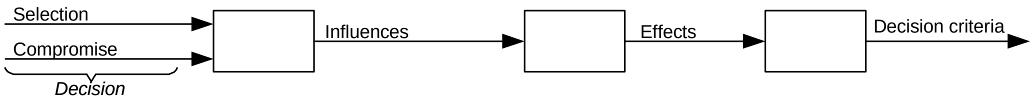

Figure 5. System architecture of proposed method.

The objective in taking decisions is to proceed towards defined objectives. Decision criteria are figures of merit of the extent to which the outcome of a decision fulfills these objectives. Two characteristics of the decision criteria are important. First, the decision maker wants to know the expected value of the decision criteria. Second, he/she wants to know the likelihood of negative deviations from the expected value. This is termed risk and is a result of the uncertainty inherent in the system. Decision criteria are defined as follows.

- *Decision criteria are figures of merit that indicate the extent to which the outcome of a decision fulfils the decision maker's objectives. Decision criteria are characterized by their expected value and their risk.*

Effects carry the decision criteria. In other words, effects are events in the broadest sense that can be measured in terms of the decision criteria. A decision generally results in a multitude of effects. It is assumed in this context that the effects are independent of each other. The effects do not influence each other. We define effects in the following fashion.

- *Effects are events that are caused by the decision and that are measurable in terms of one or several of the decision criteria.*

In order to capture and quantify effects, we classify effects based on their probability of occurrence and the number of originating decisions. The resulting taxonomy of effects is shown in Figure 6.

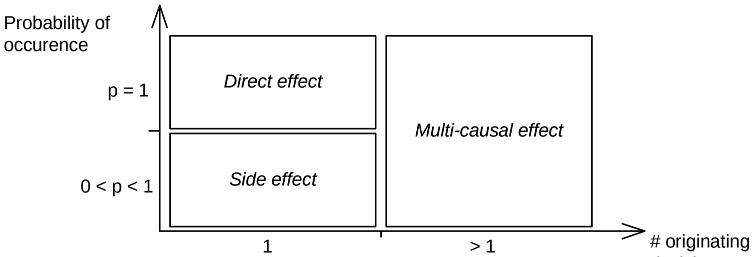

Figure 6. Taxonomy of effects.

Direct effects are the simplest type of effect. Their occurrence is certain. The occurrence of side effects on the other hand is uncertain and their probability of occurrence is therefore smaller than one. Multi-causal effects are the most complex as they cannot be traced back to a single decision. Multi-causal effects are very often neglected in decision analysis as they are only apparent if the interrelation of multiple decisions is considered. Multi-causal effects are, however, of increasing importance because they include the effects related to complexity management.

The effects of a decision are not always the same. Their magnitude in terms of the decision criteria and their probability of occurrence depend on influences. As previously mentioned, influences include both control and disturbance variables. An influence may take multiple values depending on how the decision is made. This value may also be unknown to the decision maker at the time of decision making. The decision for a particular alternative thus always implies the determination of the values of the influences. A decision is thus connected to the definition of influence/value sets, regardless of whether single influences are controllable or uncontrollable and known or unknown at the time of decision making. We define influences as follows.

- *Influences are factors that change the value of an effect in terms of the decision criteria and the probability of occurrence. An influence may take multiple values that may or may not be known at the time of decision. An influence may or may not be under the control of the decision maker.*

Functioning

Functioning comprises the mechanisms that fill the system architecture described in the previous subsection. An overview of the method's functioning is given in Figure 7. An alternative within a decision is described as sets of influences and respective values. The assignment of values to the influences is left to the human and his/her experience. The values may either be concrete values or unknown if the influence is unknown at the time of decision. Effects will look different depending on the values of the influences. That is why specific influence/value sets are mapped to specific occurrences of effects.

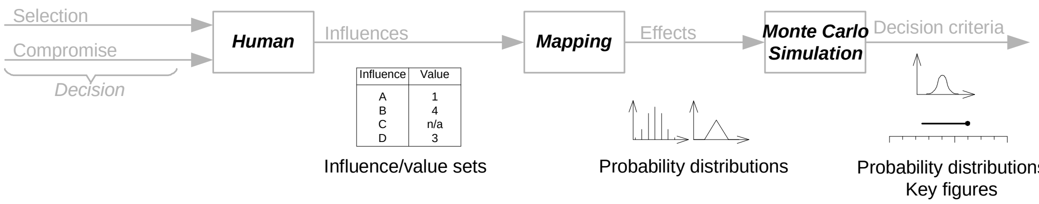

Figure 7. Functioning.

As outlined in the previous subsection, an effect needs to be measured in terms of the decision criteria. In most instances, the exact value will be uncertain. That is why all effects require probability distributions for the decision criteria that they can be measured in. The probability distribution may be discrete or continuous and may be of different types, e.g. normal or triangular distribution. Side and multi-causal effects also require information on their respective probability of occurrence. Since multi-causal effects are the result of more than one decision, they require a conversion factor to distribute the effect among multiple decisions.

In order to compare and improve several alternatives, the decision maker will want to summarize the different effects. There may for instance be five different direct effects characterized by capital expenditure for a particular alternative. Each of these effects has an associated probability distribution. The decision maker will want to calculate the overall probability distribution for capital expenditure. This can be done analytically by calculating the convolution integral among the different probability distributions. This approach is, however, very tedious to carry out. It gets even more cumbersome for side and multi-causal effects. That is why a numerical approach is used here. Unlike most other methods, Monte Carlo simulation does not obey the so-called "curse of dimensionality" [11]. The computational cost does not depend on the complexity of the system. That is why Monte Carlo simulation is applied to calculate the overall probability distribution of each decision criterion from the individual probability distributions, probabilities of occurrence, and conversion factors. This works in the following manner.

A random number generator is used to generate outcomes in terms of the decision criteria for each effect. The numbers are generated according to the respective probability distribution, probability of occurrence, and conversion factor. The outcomes of all effects are then added up to obtain the value of the decision criterion for one single experiment. This experiment is

repeated many times. If the number of experiments is sufficiently large, the frequency distribution of outcomes approaches the actual convolution integral.
The decision maker thus obtains a frequency distribution for each decision criterion as the result of Monte Carlo simulation. A frequency distribution is generally too rich in information to serve as decision basis. That is why in the last step, the probability distribution is aggregated into two key figures.

- *Expected value*: The expected value ***E[D]*** is the average outcome for an alternative in terms of a decision criterion D (Figure 8). The expected value is what a risk-neutral decision maker would be interested in.
- *Conditional Value-at-Risk (CVaR)*: As shown in the introduction, risk is a property that results out of uncertainty. Most decision makers are risk averse. It is extremely important to have a figure to measure and manage risk. Authors from finance [12, 13] have proposed the Conditional Value-at-Risk (CVaR) as a good measure of risk. The CVaR is defined as the conditional expected value of the loss under the condition that it exceeds the quantile α. For a detailed definition, we point to the above-mentioned literature. In the method proposed here, α = 50% is used. In other words, $CVaR_{\alpha}(D)$ is the expected value of all outcomes below the expected value ***E[D]*** (Figure 8).

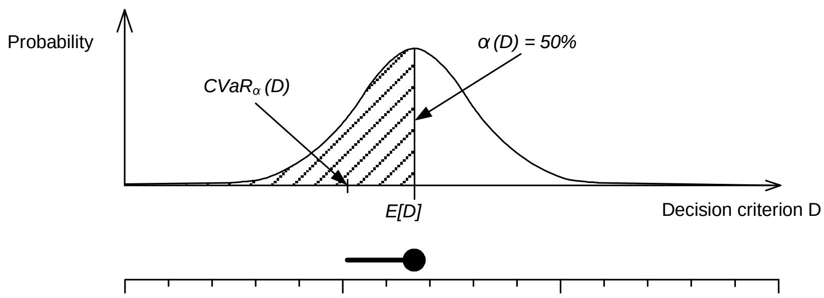

Figure 8. Expected value and CVaR.

The expected value and the CVaR are displayed graphically as shown in Figure 8. The dot is the expected value and the left end of the line is the CVaR. The length of the line thus provides a quick impression of the associated risk.
Using the expected value and the CVaR, the decision maker can rank-order different alternatives in terms of the decision criteria and thus carry out selection. The decision maker can also improve alternatives by deliberately altering the influences that are under his/her control and observing the impact on decision criteria.

Case Example

In this section, it is described how the method was implemented at a manufacturer of semiconductor equipment to support decisions in modular product development. The company applies modular product families to minimize cost while maximizing variety for the customer. The company frequently faces the decision of introducing new modules into the family in order to face up to emerging customer demands or new technologies. In the past, this decision was taken in an ad hoc manner. This often resulted in iterations in the development process, expensive prototypes, and excessive purchase prices from suppliers. Product development was altogether too expensive and too lengthy.
The Monte Carlo simulation method of multi-criteria decisions under uncertainty was therefore implemented. The steps that have been taken to implement and use the proposed method are shown in Figure 9 and described in the following.

- *Identify needs and departments*: Two main processes are relevant in new product development [14], the product development process leading from the formulation of goals and strategies to the complete product design and the realization process leading from production to the use of the product. In this context, problems arose primarily in the product development process, so we focused on this process. The departments that are involved in product development in this company are R&D, supply chain management, and process engineering. The need consisted of minimizing their effort in the development of modules.

- *Define decision criteria*: The decision criteria were clear from the needs. The workload and investments for these departments in module development should be minimized. Workload was measured in days and investments were measured in Swiss francs.
- *Identify and quantify effects*: Workshops were executed with each of the three relevant departments. In these workshops, a flipchart was used to first list all direct, side, and multi-causal effects of the decision to develop a new module. Most of the effects were those steps that are commonly associated with product development and that can be found in any engineering design textbook [15]. Numerous effects were, however, not that obvious. The multi-causal effects for instance included the life cycle management of modules, the need for an expensive PLM system, or increased stock keeping. All these effects are the result of several decisions to develop a new module. The effects were then quantified using probability distributions of the two decision criteria (workload and investment)
- *Identify and quantify influences*: Effects with a large scatter in terms of the probability distribution were addressed and influences to the probability of occurrence and the probability distribution of these effects were identified. Different values of each of the influences were identified. Probability distributions and probability of occurrence were then associated with each value. Examples of influences that were identified as being significant are the tolerances of sensors, the level of integration of mechanics, electronics, and software, or the functional range.
- *Implement simulation*: The results from the workshops, i.e. flipcharts with influence diagrams, were entered into a simple VBA software tool, which was developed for that purpose.
- *Run Monte Carlo simulations and make decisions*: The complete software tool with the simulation data was given to decision makers. In the application, the method turned out to support decision in the following manner. The quantitative assessment of alternatives in terms of expected value and the CVaR allows comparing alternatives while keeping in mind risk. Also, the method has turned out to help in improving alternatives, by taking measures to change the value of influences. Using the method, the marginal utility of these measures is immediately quantifiable in terms of the decision criteria. Once experience has been made with a larger number of decisions, it will also be possible to define standards in terms of the expected value and the CVaR that should always be satisfied. This has, however, not been done yet.

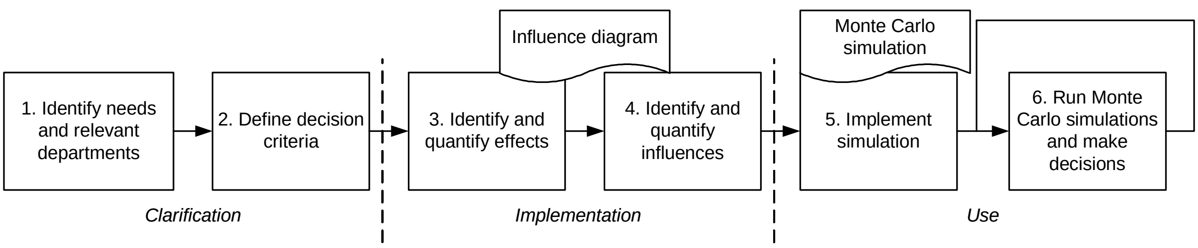

Figure 9. Method's application in case example.

Conclusions

A method to simulate decisions in terms of decision criteria has been proposed. Decision criteria are quantified in terms of their expected value and the Conditional Value-at-Risk (CVaR). The decision maker may use selection and compromise to alter the values of influences of a decision. The influences have an impact on the probabilities of occurrence and the probability distributions of a decision's effects. The overall distributions for the decision criteria are calculated from the individual distributions using Monte Carlo simulation. These distributions are then condensed into the expected value and the CVaR. The application of the method to decisions in modular product development has been shown.

In this last section, the proposed method is evaluated against the requirements set out in the background section. Avenues for future research are given.

Evaluation

A method suitable for decision support should fulfill the following two functions.

- *Allow the assessment of alternatives based on multiple decision criteria*: In the proposed method, multiple decision criteria may be defined. Different alternatives can then be compared based on how they perform in terms of the expected value and the associated risk of these decision criteria. **à** Fulfilled
- *Help improve alternatives according to the decision criteria*: Alternatives can be improved by deliberately taking actions to alter the influences. The impact of these actions on the decision criteria is directly measurable in terms of the expected value and the CVaR. The marginal utility can therefore be quantified and the decision maker can determine whether the marginal utility justifies the cost. **à** Fulfilled

In addition to the above functions, we also set out that the method should have the following characteristics.

- *Be quantitative*: The inputs to the method are the values of influences that are mapped to effects. Effects are characterized by probabilities of occurrence, probability distributions, and conversion factors. The overall probability distributions for the decision criteria are calculated using Monte Carlo simulation. The method is therefore highly quantitative. **à** Fulfilled
- *Incorporate uncertainty and risk*: Uncertainty is captured in the method by modeling the decision system using probability distributions. The risk in the decision criteria is quantified using the CVaR. **à** Fulfilled
- *Be industrially applicable*: The method was applied in one company using a series of workshops. No major obstacles occurred in the implementation. The method produced the desired outcome. The method has, however, only applied to one type of decision (modular product development), one company, and over a limited amount of time. In order to claim general industrial applicability, one would have to apply the method to different types of decisions, in different companies, and over an extended period of time. **à** Partially fulfilled

Avenues for Future Research

The following topics for future work could be identified in the scope of this research.

- *Identify standards for the CVaR*: As outlined before, the CVaR represents a suitable measure for the risk of a decision criterion. A key question that needs to be posed is the degree of risk that is acceptable in decisions. Obviously, this question cannot be answered in general. Still, it is possible to apply the proposed method to a specific type of decision in a company over a longer period of time. The CVaR and the actual outcomes of the decision should be continuously monitored. As a result, one should be able to define standards for the CVaR in specific types of decisions. These standards represent an acceptable amount of risk in an alternative and can be used to carry out risk management more effectively and efficiently.

- *Templates for influence diagrams*: Due to their ease of application, influence diagrams are used in the workshops to capture information on influences and effects. In the implementation we observed that it is highly useful to have a template for the type of decision at hand which only needs to be adapted to the particular company. This increases the likelihood of including all important effects and speeds up the workshops. We are currently in the process of defining templates for decisions in modular product development based on observed effects from implementation and findings from the literature.

Acknowledgments

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