THE USE OF SENSITIVITY OF OBJECTIVES FOR EVALUATION OF PARETO-OPTIMAL SOLUTIONS

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Abstract: *In multi-objective optimization problems the solution is not a single one but it is a set of so-called Pareto-optimal solutions. In order to select the one from this set some additional requirements are necessary. In this paper the use of the objectives sensitivity to the decision variables deviations is proposed as a criterion for the best solution. The proposed approach is presented in form of a consistent procedure in which polyoptimal solutions are clustered and their neighbourhood is investigated regarding to the objective behaviour. The knowledge obtained in course of the investigation aids selection of robust solutions and is useful in the rational allocation of tolerances. The emphasis is put for visualisation of the dependencies. As an example of application multiple-disk clutch brake optimization problem is described.*

1. INTRODUCTION

The design can be interpreted as a process of taking of optimal decisions during design problem solving. Because requirements for designed object are multifold and often contradictory, so the best solutions must be result of a compromise. These are called the Pareto-optimal solutions [7].

To make a rational choice of one solution from among a set Pareto-optimal ones requires assumption of some additional conditions. The important one is the sensitivity to changes of values of the design (decision) variables. It should be noted that these variables always take random values in the range of permitted variation i.e. the tolerance limit [1, 4].

And so, the sensitivity of the criterial variables to the deviations of the decision variables should be one of the most important pointers to decision making and to the tolerance determination.

It is commonly known that the issue of tolerance allocation is considerably related to the designed object manufacturing cost as well as its reliability. Generally, the more tight tolerances the bigger cost [5, 8].

In this context it seems reasonable to develop tools which enable better insight into the behavior of the designed object properties in the vicinity of Pareto-optimal solutions.

Consequently, in this paper a method of finding out and evaluating the poly-optimal solutions is presented. The evaluation is carried out with regard to sensitivity to deviations of design variables. An example of the method application is also shown.

2. OUTLINE OF THE METHOD

The proposed approach for analysis and evaluation of polyoptimal solutions is an extension of the study on representations of relations between variables occurring in engineering calculations [2]. It consists of the following steps:

- defining of a formal model for the design optimization problem in question {determination of decision variables (DV) and their ranges as well as objective functions (OF), setting up suitable mathematical relationships for the designed object},
- carrying out multi-objective optimization (finding out solutions for multi-input – multi-

output problem), which results in a set of Pareto-optimal solutions,

- clustering of obtained solutions in order to determine the representative ones on the grounds of the identified similarities,
- designing of experiments in centers of the clusters,
- establishing of meta-models (usually by means of the response surface methodology, RSM) which describe the relations of the type OF=f(DV),
- statistical verification of the response surface equations (RSE),
- creation of Pareto charts (for every particular criterion and for every optimal solution) and response profiles (for all criteria and for every optimal solution),
- carrying out sensitivity analysis that is investigate of effects of variation of design parameter values on design objectives (i.e. optimization criteria),
- establishing charts of RSE gradients (for every particular criterion and in all optimal solutions) and sensitivity charts (it shows the behavior of all objective criteria in all optimal solutions, thus it gives a comprehensive view on the whole optimization problem),
- interpretation of information gained heretofore,
- selection of the solution with the least sensitivity to variation of design variables. It is the best solution in the sense of the design object robustness.

Figure 2 shows flowchart of the procedure.

3. EXAMPLE OF APPLICATION

3.1. The problem

To demonstrate application of the proposed method let us consider optimal design of multiple-disk clutch brake (fig.1). Mathematical model of this assembly is taken from the report [3].

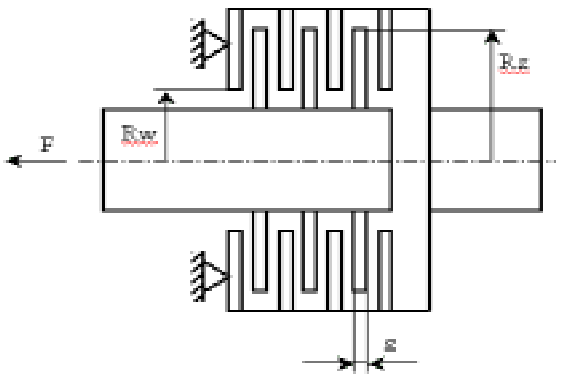

Fig. 1. *The scheme of multiple-disk clutch brake*

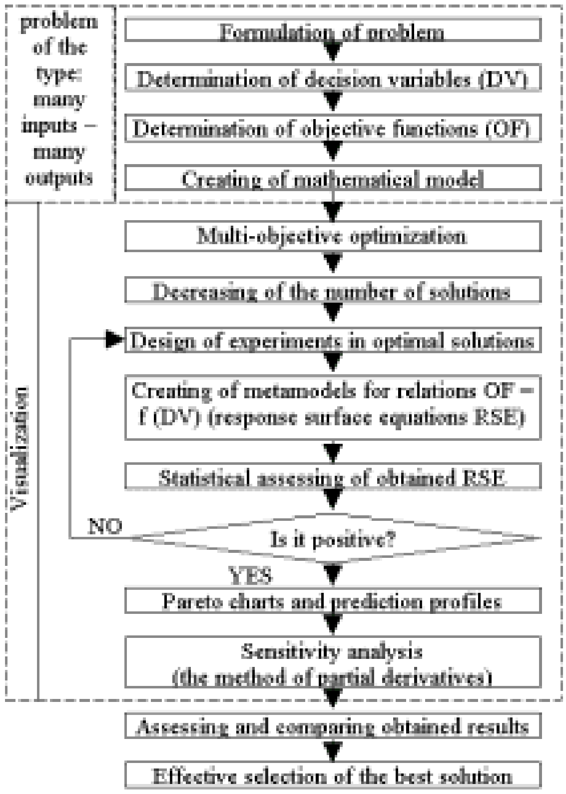

Fig. 2. *The procedure of selection of the best solution from the Pareto set of solutions*

Five decision variables were determined, which are shown in Tab. 1. Two objective criteria were: *mh* – brake disks mass [kg] and *th* – time of stopping brake [s].

The purpose of investigation is finding such values of design variables, for which (i) the optimization criteria *mh* and *th* reach minimal values and (ii) investigation of behaviour of the criteria in the neighbourhood of the Pareto-optimal solutions. On this basis the selection of the (best) solutions are found. Thus, an objective function f is to be determined:

$$f_i(\mathrm{mh}, \mathrm{th}) \rightarrow \min \qquad (1)$$

that is subjected to the decision variables:

$$R_w, R_z, g, F, Z \geq 0 \qquad (2)$$

which vary in ranges shown in Table 1.

Table 1. *Ranges and type of decision variables*

Symbol	Name	Type	Range
Rw [mm]	inner diameter of discs	continuous	60 – 80
Rz [mm]	outer diameter of discs	continuous	90 – 110
g [mm]	thickness of discs	continuous	1 – 3
F [N]	actuating force	continuous	600 – 1000
Z [-]	number of friction surfaces	discrete	{2 … 10}

3.2. Multi-objective optimization

For solving the multi-criterial problem a genetic algorithm for multi-objective optimization (MOGA) was used [6]. The search was carried out for the following parameters of the genetic algorithm: number of generations – 300, population size – 50, rate of mutation – 0,3. Moreover, blending crossover and Pareto-optimal sorting were used. Fig. 3 depicts Pareto front (fitness 1 = *mh* (mass); fitness 2 = *th* (time)) for two conflicting objective functions.

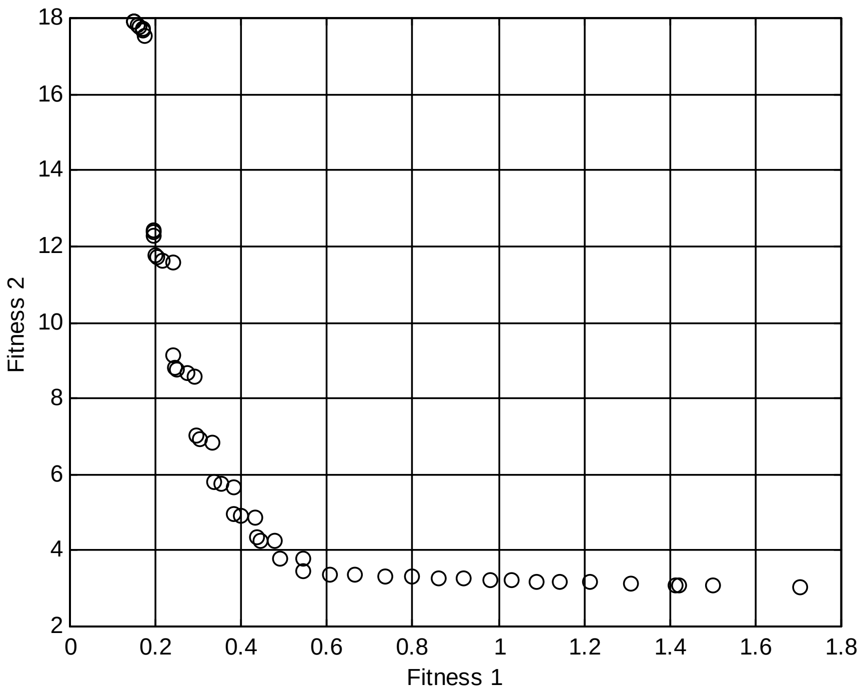

Fig. 3. *Pareto front obtained after using MOGA (Fitness 1 = mh; Fitness 2 = th)*

3.3. Cluster analysis

The number of polyoptimal solutions was arbitrarily fixed equal to 50. It was found that some solutions are very similar to some other ones, therefore investigating of each individual solution is unnecessary. In order to simplify the subsequent steps of analysis of solutions their number was reduced by applying a clustering method. This resulted in 3 representative solutions (fig. 4).

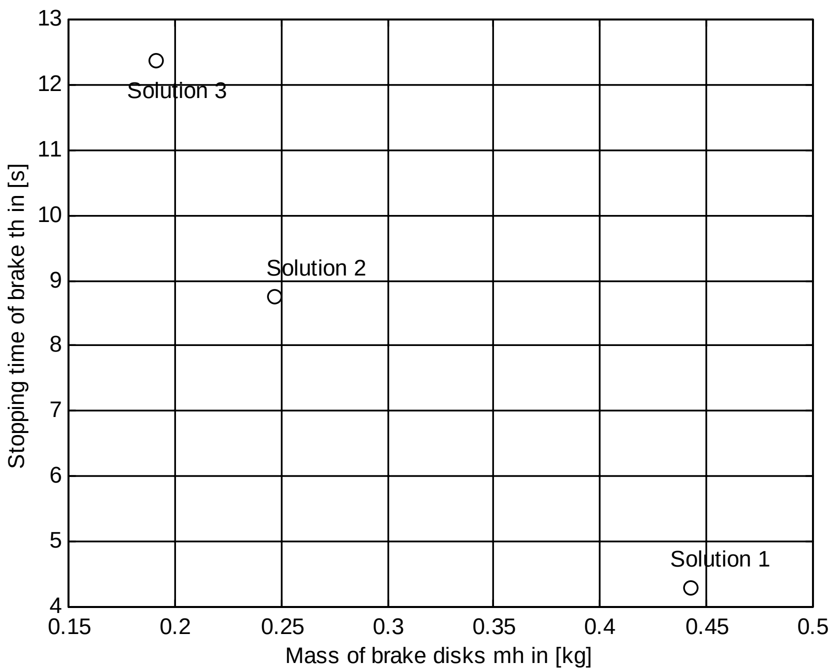

Fig. 4. *Representative optimal solutions accepted for investigation*

3.4. Design of experiments, creation of meta-models and their evaluation

In the vicinity of each of the three representative optimal solutions numerical experiment was designed. It was aimed at finding a function approximating relations between (OF) and (DV). The experiments (central composite designs) were prepared in such a way that the centre of experiment coincided with the optimal solution.

For the purpose of approximation of relations OF=f(DV) polynomials of second degree were used. It resulted in 6 response surface equations (for 3 solutions and 2 criteria). The equations were assessed by means of statistical indicators such as coefficient of determination R^2 (the closer to 1 the better) and root mean square error RMSE (the closer to 0 the better). The results of the evaluation were satisfactory. For example, in solution 1 for objective function *mh* coefficient R^2 = 1 and RMSE = 3.1371e-6 were obtained, whereas for objective function *th* - R^2 = 1 and RMSE = 4.5636e-4. These results enabled one to pass to the further analysis.

3.5. Pareto charts and prediction profiles

Positive statistical evaluation of the approximating response surface equations (RSE) justifies to proceeding to the next step of the procedure, i.e. making a local investigation of the obtained results. This step consists of setting up of Pareto charts. They enable one a deeper insight into the relations of the type OF=f(DV) in the neighborhood of a particular optimal solution. For the sake of limited scope of the paper only one Pareto chart is shown in fig. 5. In original report this type of graphs were created for all criteria in all optimal solutions.

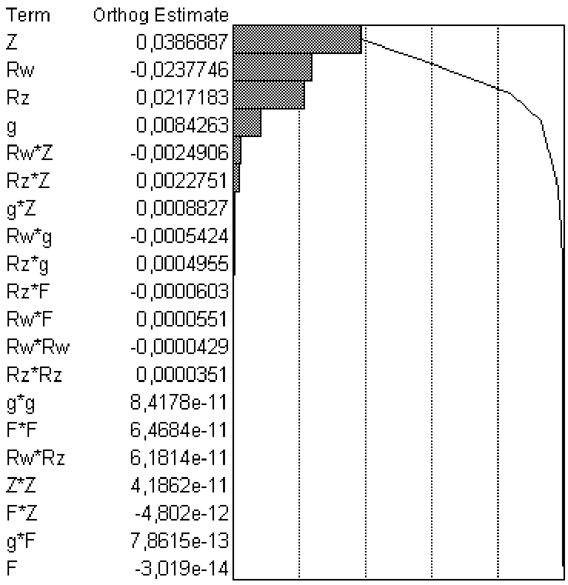

Fig. 5. *Pareto chart for criterion of mass (mh) for optimal solution number 1*

It is clear from that chart that mass of brake is the most sensitive to the number of friction surfaces *Z*, what in some way agrees with intuition. However, other variables like *Rw* and *Rz* are also important. So, small changes in their values can cause considerable variation of output *mh*.

Another useful tool are prediction profiles. They enable one to trace the behavior of all objective functions in selected optimal solution. Fig. 6 shows the prediction profiles for two criteria: *mh* (mass) and *th* (time) in optimal solution number 1.

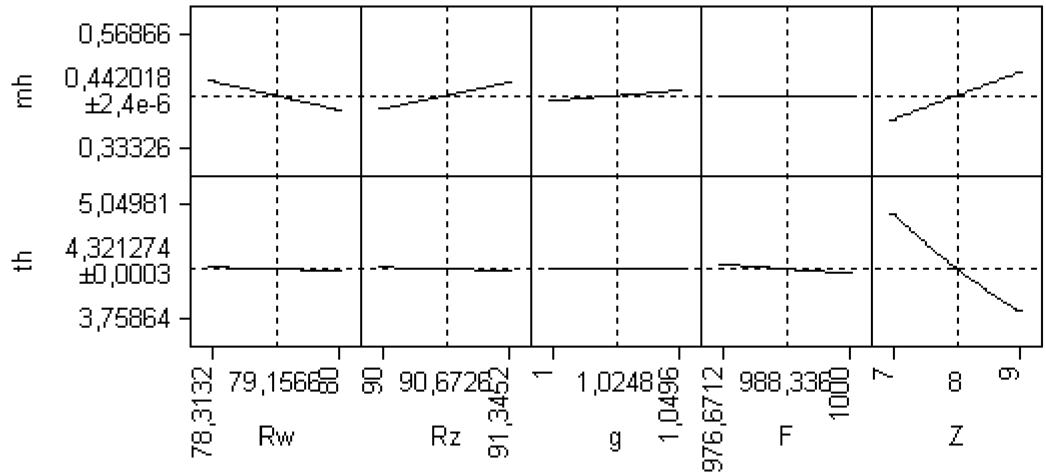

Fig. 6. *Prediction profiles for first optimal solution*

The profiles visualize behavior of both criteria in the vicinity of the solution number 1. It can be seen that the objective function *th* (stopping time of brake) depends essentially on the number of friction surfaces *Z*. The other parameters do not have any influence on this criterion, because the slope of the profiles for *Rw, Rz, g* and *F* is negligible. Sensitivity of objective function *mh* (mass of brake disks) to the decision variables is different from the former one. The most influential variable is the number of friction surfaces *Z*, but other parameters such as *Rw* (inner diameter of disks), *Rz* (outer diameter of disks) and *g* (thickness of disks) are also important.

3.6. Sensitivity analysis (gradient diagrams)

Response surface equations RSE enable one to calculate the gradients of objective functions using a method of partial derivatives. The vector, which consists of partial derivatives is called a gradient. Its components determine the strength of influence of each design parameter. Fig. 7 shows the gradient diagrams for the objective function *th* for all three representative optimal solutions.

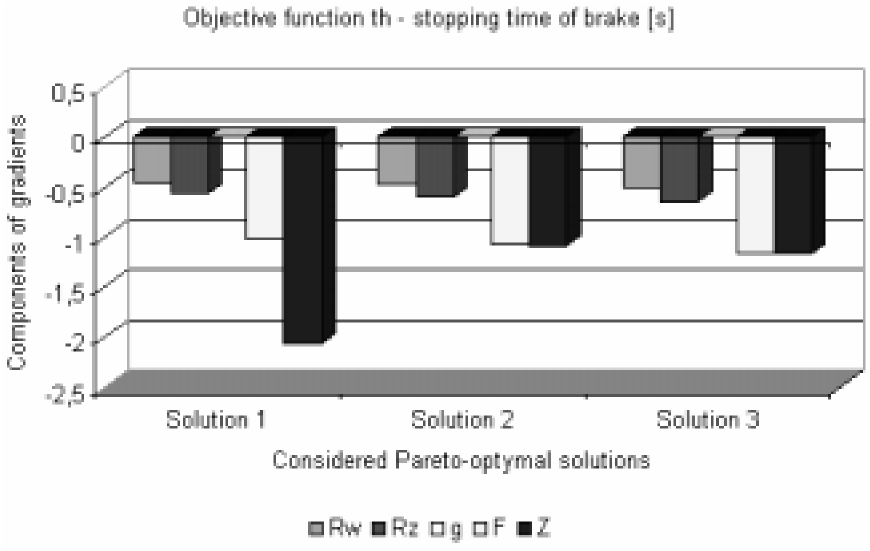

Fig. 7. *Diagram of gradients for th criterion*

This diagram displays clearly for which one of the three solutions the criterion value is most stable that is the least sensitive to the variation of values of decision variables. The thickness of discs *g* does not affect stopping time *th* in any optimal solution, whereas the number of friction surfaces *Z* is always the influential parameter. The design variables *Rw, Rz* also deserve consideration. It is also worth to note that by a variation of the actuating force *F*, one can control the stopping time *th.*

3.7. Chart of influences

Charts of influences depict the dependences between all criteria (objective functions) and all decision variables for all considered solutions. Thanks to this they give a comprehensive view on the objective function sensitivity.

It can be seen by inspection of the chart on fig. 8 which one of the two objectives is particularly sensitive to variations of the design variables. For the objective function *mh* these are variables *Rw* and *Rz* and the variable *Z* for all three representative solutions, and the variable *Z* only for the objective *th.* On the other hand the *mh* criterion is weakly sensitive to the variable *F* whereas the *th* criterion sensitivity to *Rw, Rz, g* is negligible. The important feature is that decreasing the number of friction surfaces *Z* results in decreasing the brake mass (objective function *mh*) and, simultaneously, in increasing the stopping time (objective function *th*). This phenomenon appears in all three solutions.

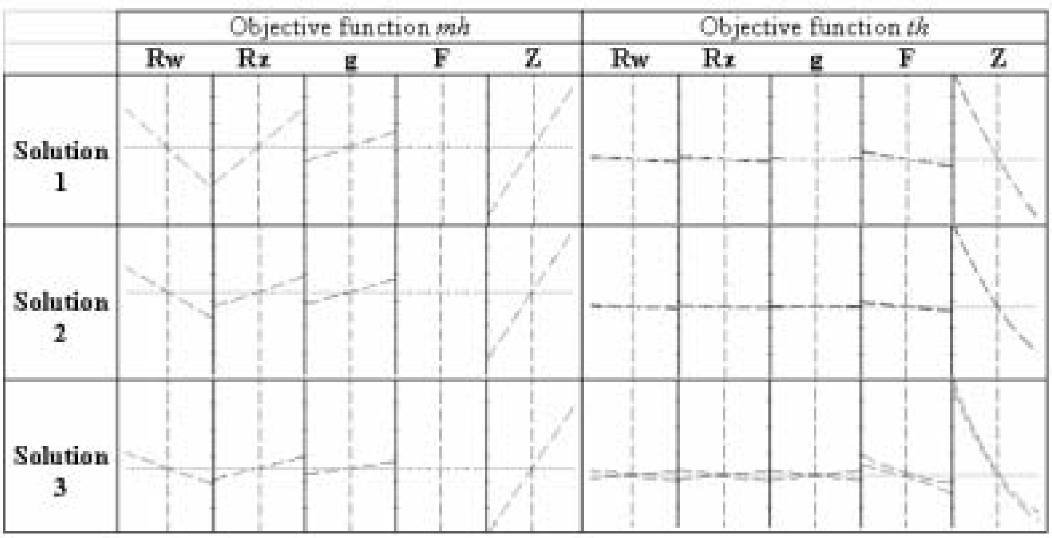

Fig. 8. *Map of influences for the case of multiple-disk brake*

4. CONCLUSIONS

The approach to optimization process presented in this paper can be summarised as follows:

- The essence of the proposed approach is presentation of results in the graphical form. It results in a better view on a considered problem and, consequently, it makes the decision-making process more user-friendly.
- The method offers a possibility of a more thoughtful insight into sensitivity of Pareto-optimal solutions to variation of decision variables, which is very important factor of solution evaluation.
- The knowledge about sensitiveness of objective functions to changes in the decision variable (DV) values enables one to allocate the tolerance limits in a more rational way. For example, if the objective function reacts to changes of a DV weaker, the tolerance limit can be broadened, which makes the manufacturing costs lower. Conversely, if the objective function sensitivity is great then it is necessary to tighten tolerances of the appropriate variables.

The method presented in this paper is an intentional composition of a number of advanced computerized techniques. They have been co-ordinated in form of a consistent procedure shown in figure 2. As a consequence of this the synergy effect is expected. The individual, separate, application of those techniques, without the co-ordination, would not result in such an effect.

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