INTERNATIONAL DESIGN CONFERENCE - DESIGN 2008
Dubrovnik - Croatia, May 19 - 22, 2008.

DESIGN CATALOGUES FOR MECHANISM SELECTION

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Keywords: mechanisms, catalogues, design optimisation, closed curves

1. Introduction

Linkage mechanisms have many uses in design and engineering. They can be used to generate intricate motions, with applications including assembly tasks [Reinhart & Cuiper 1999, Lee & Hervé 2007], packaging operations [Hicks et al., 2006, Sirkett et al. 2007], and robotics [Dai & Wang 2007]. Such motions are usually closed curves and often they are purely planar.

Mechanisms are preferable to their main (mechanical) alternative which is the use of cams since they have better performance in terms of wear and subsequent loss of accuracy. The main drawback is the fact that they are difficult to design in the first place. There are normally several degrees of freedom involved, and it is difficult for the designer to gain an easy appreciation of the effects of these separately and in combination.

The aim of this paper is to explore a means for creating computer-based catalogues of mechanisms which store the parameters for the mechanism along with its output motion. Catalogues have been used successfully in other areas of design, notably component selection [Vogwell & Culley 1991], and, with the increasing rise of web-based catalogues from component suppliers, they are used with familiarity by the modern designer.

The main obstacle to be overcome is how to describe the output path in a form that is suitable for storage in a catalogue. This is achieved by the use of Fourier analysis and the basic theory for this is given in section 2. Some of the Fourier coefficients can be given physical meaning and, as discussed in section 3, this allows an output path to be “normalised”. Section 4 explains how a catalogue can be set up and used, and section 5 indicates how any selection from the catalogue can be made more optimal and how several selections can be used to explore the local design space.

2. Theory

One application of mechanisms is in the creation of specialised motions [Molian 1982, Brix et al. 2006]. These are often in the form of closed curves which can be used for various applications including the movement of product or the implementation of complex assembly operations. As an example, consider the simplest mechanism which is the four bar chain. Figure 1 shows a “stick” diagram of the typical one such mechanism. There are three moving links: the crank, the coupler and the driven; the fourth link is the base which is here regarded as being fixed. As the crank is rotated, so the other two moving links are set in motion. The coupler link is shown with an offset point which is the end-effector for the system in this case. During the motion, the closed path, shown as a sequence of points, is traced out. One portion of this motion is approximately a straight line segment. So with some repositioning, the mechanism can be used to push product along a horizontal conveyor and then return by passing over the top of the next product that appears.

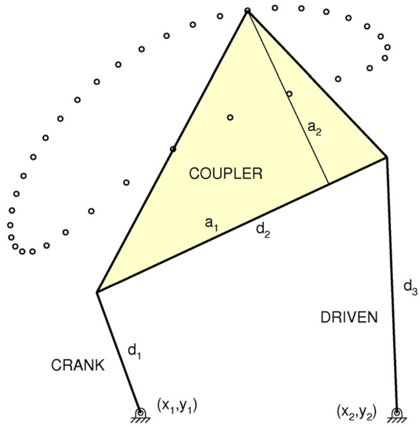

Figure 1. Four bar mechanism with an output path and its parameters

Most mechanisms that appear in industrial machines are essentially two dimensional and so it is also assumed here that the path is planar. In order to be able to classify and compare output paths, it is necessary to have some means for describing them numerically. One way to do this is in terms of a Fourier representation [McGarva & Mullineux 1993] and it is this approach that is reviewed here.

The complex plane is used for modelling the output path. This means that the typical point on the path is regarded as a complex number of the form

$$z(t) \ = \ x(t) + iy(t) \tag{1}$$

where i is the square root of (-1) and t is a parameter which is regarded as being related to the angle of the driving crank. For simplicity, it is assumed that the parameter is normalised so that t passes between 0 and 1 as a single full cycle of the mechanism takes place.

The standard Fourier theory (assuming that $z(t)$ satisfies Dirichlet's conditions) [Apostol 1957] says that $z(t)$ can be represented as the doubly infinite series

$$z(t) \ = \ \sum c_m \exp(2\pi imt) \tag{2}$$

where the series is summed between $m = -\infty$ and $m = +\infty$, and the constant coefficients c_m are normally non-real. These coefficients are given by

$$c_m \ = \ \int \exp(-2\pi imt)\, z(t)\, dt \tag{3}$$

where the integral is over a full cycle, that is between t=0 and t=1. For convenience, the coefficient c_0 is called the fundamental, coefficients c_1 and c_{-1} are regarded as forming the first harmonic, coefficients c_2 and c_{-2} the second harmonic, and so on.

Normally the output path is not easily expressed as an explicit function. Instead it is available as a sequence of points along the curve. This is not a problem as the integrations can be carried out numerically. Suppose N points are given

$$z_0, z_1, z_2, \ldots, z_{N-1} \tag{4}$$

where, when necessary, this sequence is regarded as being circular so that z_k is the same as z_{k-N}. If it assumed that the points correspond to equally spaced values of the parameter t, then the step length for numerical integration is (1/N) and the trapezium rule provides the following approximation for the Fourier coefficients (using the fact that the path closed)

$$c_m \ = \ (1/N) \sum z_k \exp(-2\pi imk/N) \tag{5}$$

where the sum is between $k = 0$ and $k = N$-1.
In the case when m=0, it is seen that the fundamental coefficient is simply the average of all the points. It thus represents their centroid. This means that $z(t) - c_0$ represents a closed curve whose centroid lies at the origin of the complex plane. Now consider the first harmonic terms, specifically the function

$$z_1(t) \;=\; c_1 \exp(2\pi i t) \;+\; c_{-1} \exp(-2\pi i t) \tag{6}$$

With some manipulation, this can be rewritten in the form

$$z_1(t) \;=\; \exp(i\alpha)\,[\,(r_1 + r_{-1})\cos(2\pi t + \beta) + i(r_1 - r_{-1})\sin(2\pi t + \beta)\,] \tag{7}$$

where

$r_1 = |c_1|$
$r_{-1} = |c_{-1}|$
$\alpha = [\arg(c_1) + \arg(c_{-1})]/2$
$\beta = [\arg(c_1) - \arg(c_{-1})]/2$

In the last expression for $z_1(t)$, the term inside the square brackets represents an ellipse. The centre of this ellipse lies at the origin of the complex plane and its semi-major and semi-minor axes have lengths $(r_1 + r_{-1})$ and $(r_1 - r_{-1})$. The effect of the multiplying complex exponential $\exp(i\alpha)$ is to rotate the ellipse through an angle α anticlockwise. This is shown in figure 2. If r_1 is larger than r_{-1}, then the semi-major axis has positive length and the ellipse is drawn anticlockwise as t varies from 0 to 1. If r_1 is smaller than r_{-1}, then the curve goes clockwise. If $r_1 = r_{-1}$, then the ellipse collapses to a straight line lying at angle α to the real axis.

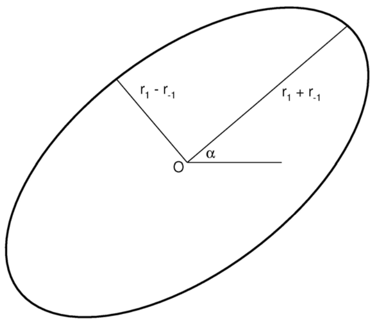

Figure 2. Ellipse for first harmonic terms

Similar considerations apply also to pairs of higher harmonic coefficients. The expression

$$z_m(t) \;=\; c_m \exp(2\pi i m t) \;+\; c_{-m} \exp(-2\pi i m t) \tag{8}$$

also represents an ellipse centred at the origin. The main difference is that this is traced out m times as the parameter t passed from 0 to 1.
Figure 3 shows an example of the decomposition of a closed curve. The main part of the figure is the original curve itself. The parts numbered 1 to 5 are the results of forming the partial sums up to first through to the fifth harmonics. It is clear that the original curve is reasonably accurately reproduced with just three harmonics. This is also seen in table 1 which lists the complex Fourier coefficients of the fundamental and first five harmonics. It is seen that the (absolute) values of these tend to decrease as the higher harmonics are reached.

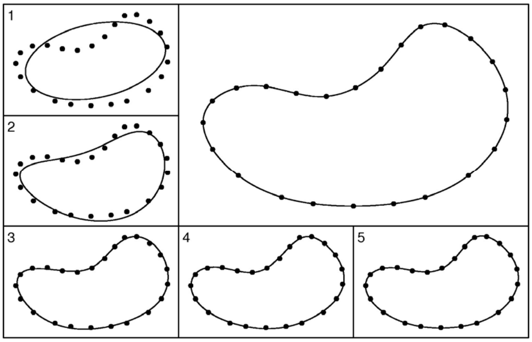

Figure 3. Decomposition of a closed path curve

Table 1. Fourier coefficients for closed path in figure 3

m	c_m
-5	-0.051 + 0.016i
-4	0.110 - 0.070i
-3	-0.291 - 0.059i
-2	0.644 - 0.080i
-1	-0.865 + 1.983i
0	0.572 - 0.603i
1	0.055 - 6.787i
2	-0.418691 - 1.297i
3	0.316 + 0.433i
4	-0.076 + 0.115i
5	-0.027 + 0.069i

3. Physical significance

It is possible to assign geometric significance to the Fourier coefficients for the fundamental and the first harmonics. This enables some normalisation of the Fourier coefficients, and hence also of the path itself, to be carried out. Firstly, as noted in the previous section, the fundamental coefficient c_0 simply represents the centroid of the original path. All this signifies is the position in the plane where the path lies: it does not affect the actual shape. Replacing coefficient c_0 by zero preserves the shape of the curve but moves its centroid to the origin of the complex plane.

Now consider the first harmonic coefficients c_1 and c_{-1}. Together these correspond to an ellipse in the complex plane whose major axis is at angle $\alpha = [\arg(c_1) + \arg(c_{-1})]/2$ to the real axis. If each Fourier coefficient is multiplied by exp(-iα), then the whole path is rotated about the origin and the major axis for the ellipse of the first harmonic is brought along the real axis. This leaves the shape of the path unchanged but brings its "widest part" parallel to the real axis.

Of the coefficients, c_1 and c_{-1}, it can be assumed that the first of these has the larger absolute value. If this is not the case, then all pairs of coefficients c_m and c_{-m} can be interchanged: this is equivalent to changing the sign of the parameter t and hence does not change the shape of the path. Now divide all the Fourier coefficients by $|c_1|$. This has the effect of applying a scaling factor to the entire path curve: it does not affect its shape, only its overall size. The value of $|c_{-1}|$ is a measure of how nearly circular the path is, with smaller values corresponding the more circular case.

The first harmonic coefficients now have the form $\exp(i\beta)$ and $|c_{-1}|\exp(-i\beta)$. Changing the parameter t to $(t + \beta/2\pi)$ has the same effect as multiplying each coefficient c_m by $\exp(-im\beta)$. This change of parameter again does not affect the shape and allows the coefficients c_1 and c_{-1} to both be taken as real, with the former equal to unity.

In this way, normalisation can be undertaken which eliminates the effects of:

- position of the path in the plane
- rotation of the path about its centroid
- direction in which the path is traced out
- size of the path
- where the path starts being traced out

Of these, only the first second and fourth relate to the actual geometry; the other two only change the form of the parameterisation. Figure 4 shows the effect of normalising an example path.

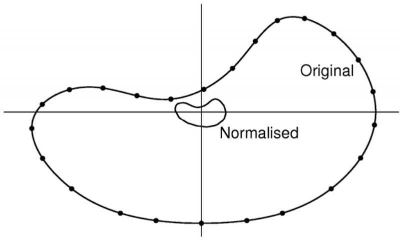

Figure 4. Normalisation of a closed path curve

4. Catalogues

For a given topology, the form of the geometry of a mechanism is essentially parametric. A four bar mechanism depends upon nine parameters. These are shown in figure 1: two pairs of the parameters form the coordinates of the fixed pivot positions, three specify the lengths of the links, and the final two determine the position of the end-effector with respect to the coupler link.

This parametric form allows catalogues of mechanisms and their output motions to be created. The procedure is as follows and relies upon having some means of computer-based simulation of a mechanism. For a given topology, the simulation is carried out iteratively, at each stage changing the parameters so that they run over a large range of values. Some choices of parameters provide a mechanism that cannot be assembled or that cannot be fully cycled: these are ignored. If the operation is successful, then the output path is recorded as a sequence of points corresponding to equal steps of the driving link. The Fourier coefficients of this output motion are then determined. The catalogue is created (as a text file) by listing the parameters of the mechanism (together with a flag value giving its type) alongside the Fourier coefficients of its motion.

The size of the catalogue can be reduced by using the normalisation idea. For any mechanism, once the Fourier coefficients of its motion are found, they are normalised. The corresponding normalisation, for position, orientation and size, is also applied to the parameters of the mechanism. If this is done, there is no need to explore all changes in the parameters. The position of two of the fixed pivot positions of a mechanism of a given topology may as well be kept fixed as varying them (keeping all the others fixed) merely affects position, orientation and size.

To use the catalogue, the following procedure is used. Given a required output motion, specified as a sequence of points, its Fourier coefficients are firstly determined. These are then normalised and then compared with values in the catalogue. The comparison is to take the differences between corresponding values, form the squares of these and take the square root of their sum (thus forming a Euclidean distance). Those entries are found in the catalogue which give the smallest values of this measure. Once found, the reverse of the normalisation applied to the desired output is applied to the best mechanisms to "unnormalise" them. It has been found beneficial to implement this search process so that the ten (or so) best mechanisms are shown to the user. This means that he/she can see the alternatives and make an informed choice, possibly based on secondary considerations.

The central part of figure 5 shows a desired motion path. Around it are shown some example mechanisms obtained by the above process which attempt to generate this motion. They have varying degrees of success. This desired motion is in fact difficult to obtain precisely as the path is re-entrant.

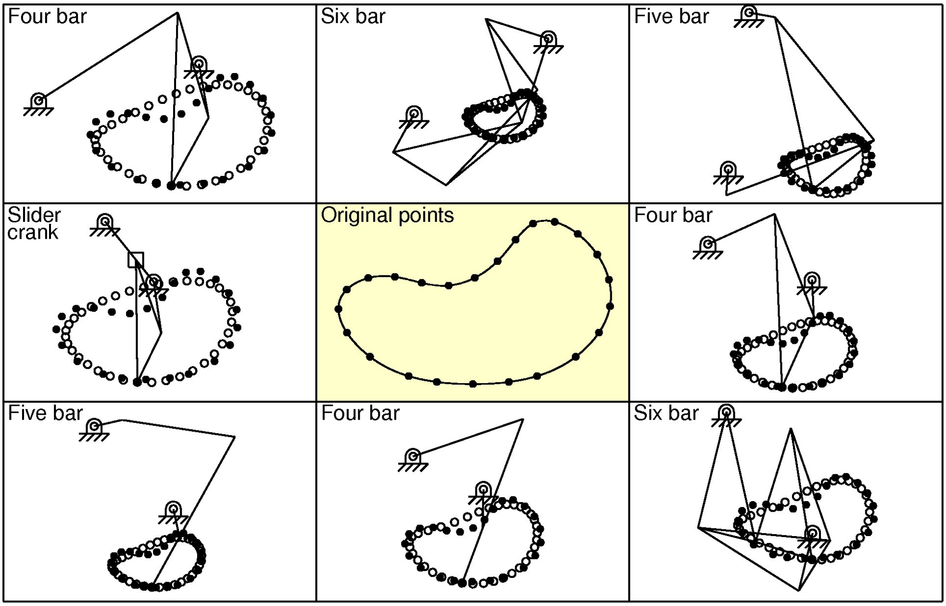

Figure 5. Mechanisms to generate a given path

5. Improvement in motion path

The catalogue idea is really intended as giving initial design solutions from which better ones can be developed. In this way, it acts as a computer-based version of the more traditional atlases of standard mechanisms (e.g. [Chironis 1966]). However, in practice, it has been found that usually at least one result obtained from a catalogue search is good enough to be used directly.

If there is a need to improve a mechanism to obtain a better match to a desired path, there are several ways in which this can be done. The simplest is to allow the mechanism parameters to vary by small amounts and see what effect this has upon the output motion. Since the measure of the similarity of two sets of Fourier coefficients is available, it is possible to treat this as a function of the mechanism parameters and adopt an optimisation strategy. In this way the catalogue is used to provide "seed" mechanisms to act as starting points for the optimisation process.

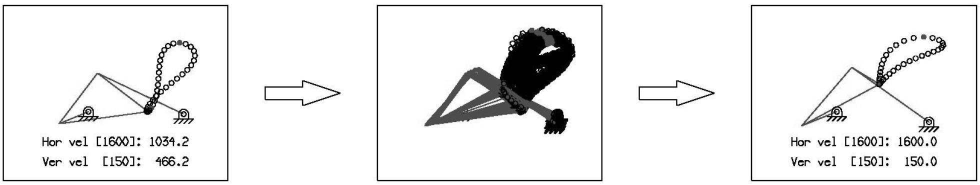

Figure 6. Improvement of a transfer mechanism

In other cases, it may be necessary to add criteria beyond simply the motion profile. Figure 6 shows a case study involving a transfer mechanism to take product from one conveyor and place it on another moving at right angles to it. There is a speed difference in the conveyors so that the first moves at approximately 11 times the speed of the other. The motion suggests the need for a roughly triangular path. The left side of figure 6 shows a mechanism obtained by a catalogue search. However, the actual path is less critical than the need to match the velocity with that of the relevant conveyor during picking and placing. Specific speed requirements are specified at the two points in the path shown as filled dots. To achieve these, an optimisation process is started in which the mechanism parameters are varied with the aim of reducing the speed differences to zero. This process is shown in the central part of figure 6, and the last part shows the mechanism finally achieved. Here the path has changed considerably, but the speeds match exactly those required.

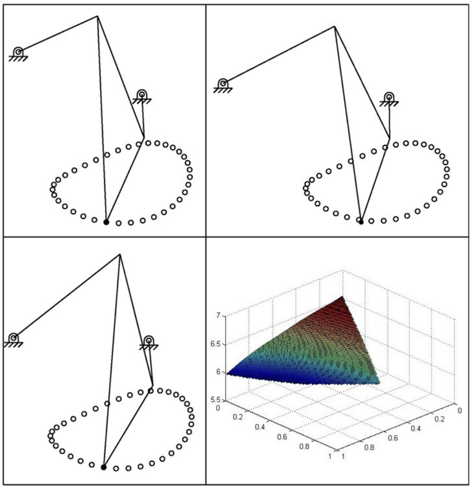

Figure 7. Three mechanism instances and surface of performance of interpolated mechanisms

An alternative approach is to treat the results from a catalogue search as "instances" of possible mechanism design and then to use these as a means for investigating the local design space [Singh et al. 2007]. This requires access to some form of parametric modeller and simulator for mechanisms. A number of mechanism instances, say three, are taken directly from the catalogue. Interpolation between these is then performed so as to create a range of additional instances. Each new interpolant is created by forming its parameters as the same weighted average of parameters from the instances. The resultant path is then compared to the desired path and the measure of its closeness obtained. A surface plot of the results can be produced for visualisation. An example is shown in figure 7. The three instances are shown and together with the corresponding surface. Here the surface is roughly flat and slightly sloping and a minimum is appears along one edge. This suggests that this design solution is insensitive to changes in the parameters. In other case, the surface produced may be less flat suggesting that there may be problems if the real mechanism were subject to manufacturing errors. It may also happen that the surface contains some form of "ridge" separating one part of the surface from the rest. Thus suggests that the original instances lie in two different families of related mechanisms. In any case, the surface form gives information about the quality of any design solution that is chosen.

6. Conclusions

The interest in this paper is in the creation of catalogues of mechanisms and their output motions so as to provide an aid for a designer who needs to find a mechanism to produce a given path. It is seen that closed curves can be described in terms of sequences of Fourier coefficients expressed as complex numbers. Normalisation can be carried out to standardise the curve geometry so that its centroid lies at the origin and its scale and orientation are standardised.

A mechanism catalogue can then be created by running simulations of particular mechanism forms with changes of parameter, and then storing those parameters together with the Fourier coefficients of their output paths. The normalisation helps to reduce the physical size of the catalogues files. To find a mechanism for a given path, that path is normalised, compared with entries in the catalogue, the best match found, and the corresponding mechanism unnormalised.

Once one or more mechanisms are found, the match can be improved by further optimisation, perhaps with the inclusion or other requirements. Entries from the catalogue can also be used as design instances to start exploring the local design space.

Acknowledgements

The work reported in this paper has been carried out under a number of related research grants. The most recent has been funded by the Department for the Environment, Food and Rural Affairs (DEFRA) in connection with the Food Processing Faraday Knowledge Transfer Network (FPF KTN). This support is gratefully acknowledged.

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