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FOUR-BAR LINKAGE DESIGN USING GLOBAL OPTIMIZATION APPROACH

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1. Introduction

The paper discusses optimal synthesis of four-bar linkage. The general optimization problem is addressed in the form of nonlinear programming problem. The objective of this approach is to determine the optimal values of the mechanism links length, to minimize hinge forces, while the difference between the trajectory T of the arbitrary point C on the mechanism coupler link and the prescribed trajectory L has to remain within the prescribed values. The global optimization method is used in order to find the global optimal solution. The procedure uses the Adaptive Grid Refinement algorithm. This algorithm is based on identification of feasible nodes in each iteration defining the solution set. Nodes far from the current optimum are pruned from the solution. The algorithm identifies optimal regions that satisfy predefined conditions, rather than only a single optimal point.

2. Methods

2.1 Mechanical model of four-bar linkage

There are several notations useful for performing kinematic and kinetic analysis, amongst which the vector notation [Shabana 1994, Waldron and Kinzel 1999] is particularly suitable for analyzing planar four-bar mechanism depicted in Figure 1 by using symbolic manipulations [Wolfram 1996]. Thus, mechanism links are represented as vectors of length $r_i, \quad i = 1,2,3,4$, that make a closed vector loop. The link r_1 is fixed at the angle q_1; coordinates of the arbitrary point C on the coupler are denoted by x_C, y_C. The input parameter is the angle $q_2 = q_2(t)$, on which the configuration of four-bar mechanism and subsequently all other coordinates describing configuration of mechanism are dependent. To solve the kinematic, the loop closure condition needs first to be written

$$\mathbf{r}_1 + \mathbf{r}_4 = \mathbf{r}_2 + \mathbf{r}_3 . \tag{1}$$

Vector equation (1) can be rewritten as a system of two scalar equations having unknowns q_3 and q_4

$$\begin{aligned} r_3 \cos q_3 &= r_1 \cos q_1 + r_4 \cos q_4 - r_2 \cos q_2 , \\ r_3 \sin q_3 &= r_1 \sin q_1 + r_4 \sin q_4 - r_2 \sin q_2 . \end{aligned} \tag{2}$$

By squaring and adding (2) q_3 is eliminated and after some additional algebraic manipulation the angle q_4 is obtained in terms of the parameter q_2.

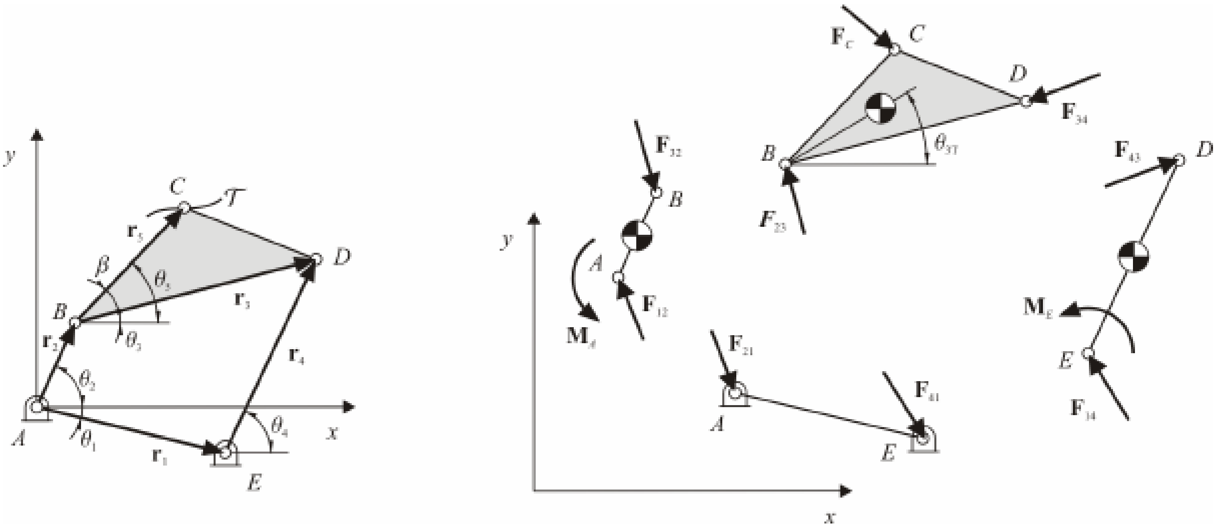

Figure 1. Notation of four-bar mechanism

Figure 1 clearly demonstrates the relation between the angles q_3 and q_5:

$$q_5 = q_3 + b \ . \tag{3}$$

The position of the arbitrary point C on coupler is then given by equation

$$\mathbf{r}_C = \mathbf{r}_2 + \mathbf{r}_5 \, . \tag{4}$$

In order to perform kinetic analysis, the discretization approach [Shabana 1994] is used. Figure 1 depicts, how mechanism links are considered as free bodies. An external force $\mathbf{F}_C$ is applied in the point C and an external moment $\mathbf{M}_A$ is applied in the point A. Dynamic equilibrium equations are developed for each body. Newton's equation describes the motion of the center of the body mass

$$\sum_j \mathbf{F}_j = m_i \, \mathbf{a}_{iT} \, , \tag{5}$$

while Euler's equation describes rotary motion of the body caused by the forces and moments acting on the body about the center of mass

$$\sum_j \mathbf{r}_{iT}^{Tj} \times \mathbf{F}_j + \sum_k \mathbf{M}_k = \mathbf{J}_{iT} \, \mathbf{a}_i \, . \tag{6}$$

In (6) $\mathbf{J}_{iT}$ represents the inertia tensor of the i-th link. (5)-(6) can be rearranged [Shabana 1994] in the following matrix form:

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ a_{71} & a_{72} & a_{73} & a_{74} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{83} & a_{84} & a_{85} & a_{86} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{95} & a_{96} & a_{97} & a_{98} & 1 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{23x} \\ F_{23y} \\ F_{34x} \\ F_{34y} \\ F_{41x} \\ F_{41y} \\ M_E \end{bmatrix} = \begin{bmatrix} m_2\, a_{2Tx} \\ m_2 \left(a_{2Ty} + g\right) \\ m_3\, a_{3Tx} - F_{Cx} \\ m_3 \left(a_{3Ty} + g\right) - F_{Cy} \\ m_4\, a_{4Tx} \\ m_4 \left(a_{4Ty} + g\right) \\ J_{2T}\, \ddot{q}_2 - M_A \\ J_{3T}\, \ddot{q}_3 - e_1 F_{Cx} - e_2 F_{Cy} \\ J_{4T}\, \ddot{q}_4 \end{bmatrix} \tag{7}$$

(1)-(4), together with (5)-(7), fully describe the four-bar mechanism motion and forces producing motion or forces that are the result of prescribed motion and should be understood in future formulations as system equations [Hsieh and Arora 1984].

2.2 Optimization problem

The majority of engineering optimum design problems [Hsieh and Arora 1984] may be written in a form of a non-linear programming problem

$$\min f_0(\mathbf{p}), \quad \mathbf{p} \in \mathrm{R}^n \tag{8}$$

subject to

$$g_i(\mathbf{p}, \mathbf{u}, t) \le 0, \quad 1 \le i \le m_1, \ t \in [0, t]$$

$$h_j(\mathbf{p}, \mathbf{u}, t) = 0, \quad 1 \le j \le m_2, \ t \in [0, t]$$

$$p_j \in \left[\breve{p}_j, \widehat{p}_j\right], \quad 1 \le j \le n.$$

In (8) $\mathbf{p}$ represents parameter or design variable vector and $\mathbf{u}$ is system variable vector. The objective function $f_0(\mathbf{p})$ is to be minimized so that it satisfies constraint functions g_i defining feasible domain D, and system equations h_j represent the mathematical model of the considered mechanical system. The solution of formulation (8) is optimal design variable vector $\mathbf{p}^*$. To perform optimal synthesis of the four-bar mechanism with the help of nonlinear programming formulation we define the design variable vector and the system variable vector as

$$\mathbf{p} = [p_1, p_2, p_3, p_4, p_5]^T := [r_1, r_2, r_3, r_4, q_1]^T, \ \mathbf{u} = [u_1, u_2]^T := [x_C, y_C]^T, \tag{9}$$

The following formulation is suitable to minimize the hinge forces in joints A, B, D, E:

$$\min \max \left|\mathbf{F}_i(\mathbf{p}, \mathbf{u}, t)\right|, \quad i = A, B, D, E, \quad t \in [0, t], \quad \mathbf{p} \in \mathrm{R}^5 \tag{10}$$

subject to

$$g_1(\mathbf{u}, t) := \left|\mathsf{T}[u_1(t), u_2(t)] - \mathsf{L}(x, y)\right| \le d_{\max}, \quad t \in [0, t]$$

$$g_2(\mathbf{p}) := (p_3 + p_4) - (p_1 + p_2) \le 0,$$

$$g_3(\mathbf{p}) := (p_2 + p_3) - (p_1 + p_4) \le 0,$$

$$p_j \in \left[\breve{p}_j, \widehat{p}_j\right], \quad 1 \le j \le 5.$$

In (10) the difference between the trajectory T and the prescribed linear trajectory L is defined as

$$\mathsf{T}\left[u_1(t),u_2(t)\right]-\mathsf{L}(x,y):=\frac{b_1x+b_2y+b_3}{\sqrt{b_1^2+b_2^2}}, \tag{11}$$

where b_1, b_2 and b_3 are constant parameters of L. Constraint function $g_1(\mathbf{u},t)$ ensures the difference (11) to be less than $d_{\max}$. Further, constraint functions $g_2(\mathbf{p})$ and $g_3(\mathbf{p})$ represent well known Grashoff conditions, that reflect the restriction on the leading mechanism links to perform only oscillatory motion. As the length of any mechanism link cannot be negative and the frame dimension being limited to a certain maximum value, the upper and lower bounds on design variables are imposed. Formulation (10) is not soluble by nowadays known methods of mathematical programming. The issue of this problem is operator **max** in objective function and time dependent constraint function $g_1(\mathbf{u},t)$. Therefore, it is necessary to transform (11) in a way to get an adequate soluble standard form. As shown in [Hsieh and Arora 1984], we involve an artificial design variable p_6 and define

$$\min \tilde{f}_0(\tilde{\mathbf{p}}),\quad \tilde{f}_0(\tilde{\mathbf{p}}):=p_6,\quad \tilde{\mathbf{p}}\in \mathrm{R}^6$$

subject to

$$\begin{aligned}
&g_1\left(\mathbf{u},t_j\right):=\left|u_1\left(t_j\right)-l_1(x,y)\right|\le d_{\max},\quad j=1,\dots,n_1\\
&g_2\left(\tilde{\mathbf{p}}\right):=\left(p_3+p_4\right)-\left(p_1+p_2\right)\le 0,\\
&g_3\left(\tilde{\mathbf{p}}\right):=\left(p_2+p_3\right)-\left(p_1+p_4\right)\le 0,\\
&g_i\left(\tilde{\mathbf{p}},\mathbf{u},t_k\right):=\left|\mathbf{F}_i\left(\tilde{\mathbf{p}},\mathbf{u},t_k\right)\right|-p_6\le 0,\quad k=1,\dots,n_2,\quad i=A,B,D,E,\quad t\in\left[0,t\right]\\
&\tilde{p}_m\in\left[\breve{p}_m,\hat{p}_m\right],\quad 1\le m\le 6,
\end{aligned} \tag{12}$$

where

$$\tilde{\mathbf{p}}=\left[p_1,p_2,p_3,p_4,p_5,p_6\right]^T \tag{13}$$

represents the extended design variable vector, while t_j, $j=1,\dots,n_1$ and $t_k, k=1,\dots,n_2$ are local maxima of constraint functions.

As the constraints in (12) are of rather complicate form, determining of local maxima t_j and t_k would be a very demanding task. Instead, discretization of interval $t\in\left[0,t\right]$ on n_3 equidistant points is used:

$$t_l=t_0+\left(l-1\right)\frac{t}{n_3-1},\quad l=1,\dots,n_3,\quad \text{and}\quad j=l,\quad k=l. \tag{14}$$

In this research the formulation (12) is solved by using global optimization method. The Adaptive Grid Refinement algorithm (AGR) procedure is applied [5]. The AGR is in essence a generalised-descent method, which works as follows. The interval to be searched for a solution is grided into *n* initial grid nodes with equivalent distance. At each node the objective function is evaluated. The nodes with the lowest objective function values are kept, while the rest are excluded from the forthcoming procedure. At the each node kept, the new nodes are evaluated on each side, at one-third the distance between the first set of nodes. The process of grid refinement continues until the stopping criterion is met, when all calculated optimal nodes are displayed. With this procedure the population of nodes in the working set is moving downhill but over multiple possible regions and directions in each iteration.

The same procedure is used for any of the design variables, however the number of grid nodes and computation effort increases exponentially by the number of design variables. The algorithm is very stable, derivative-free and could also handle discontinuities and calculations in the proximity of a complex constraint boundary.

3. Numerical example and conclusions

As an example, the optimal synthesis of hydraulic support [Oblak et al. 1998] is performed. The hydraulic support, depicted on Figure 2, is a part of mining industry equipment, considered to protect the working environment. The aim of the research is optimal design of the leading four-bar mechanism in order to ensure desired motion of hydraulic support top part with minimal transversal displacements. Transversal displacements have to be small enough to prevent collision of the support with other machinery and equipment. The kinematics of hydraulic support could be modeled with synchronous motion of the driving mechanism $FGDE$ and the leading mechanism $ABDE$. The decisive influence on motion of hydraulic support has the leading four-bar mechanism $ABDE$. Also, the hinge loads of the same mechanism are critical.

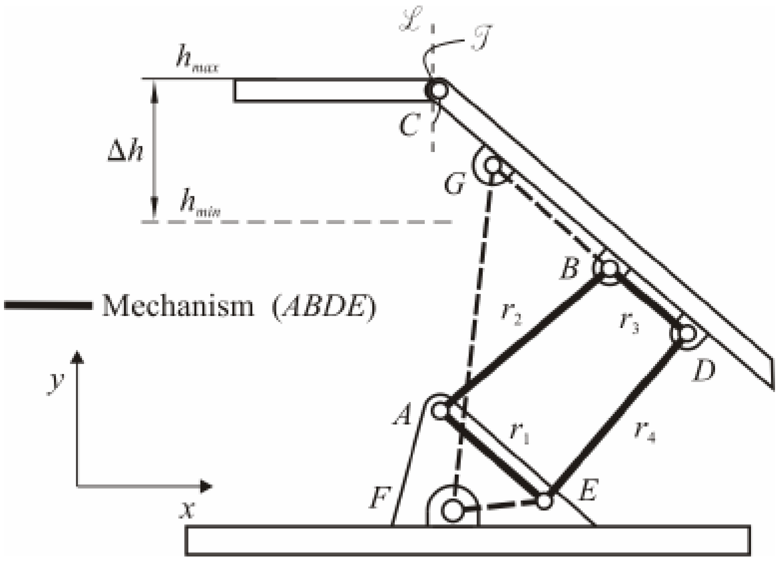

Figure 2. Hydraulic support

The optimal synthesis of this mechanism is considered to ensure the hinge forces to be as low as possible, while the trajectory of the point C should be $d_{\max} \leq 20$ mm displaced from either side of the prescribed vertical path $\mathrm{L}(x, y) := x = 65$ mm. The applied external load force is $F_C = 1178.4$ kN. The optimal leading four-bar mechanism $ABDE$ is specified by parameters vector $\mathbf{p}^* = [728.8, 1399.5, 407.4, 1391.7, -0.756]^T$ mm(rad). The trajectory of the point C and the hinge force in E are depicted in Figure 3 as solid lines. Comparing the previous existing solution (dashed lines) [6] with calculated optimal solution (solid lines) one could see a certain increase of transversal displacements from $\Delta x_{\max} = 12.2$ mm to $\Delta x^*_{\max} = 25.3$ mm. Maximal hinge forces $F^*_{A\max} = 1394.8$ kN, $F^*_{B\max} = 1376.1$ kN, $F^*_{D\max} = 1395.3$ kN and $F^*_{E\max} = 1395.0$ kN for optimal design vector $\mathbf{p}^*$ are significantly decreased for over 18 % in the critical joints D and E, comparing to the previous existing solution, where $F_{A\max} = 1062.3$ kN, $F_{B\max} = 1062.1$ kN, $F_{D\max} = 1711.6$ kN and $F_{E\max} = 1712.0$ kN. The external load is therefore more equably distributed between joints, which consequently means improved and safer design.

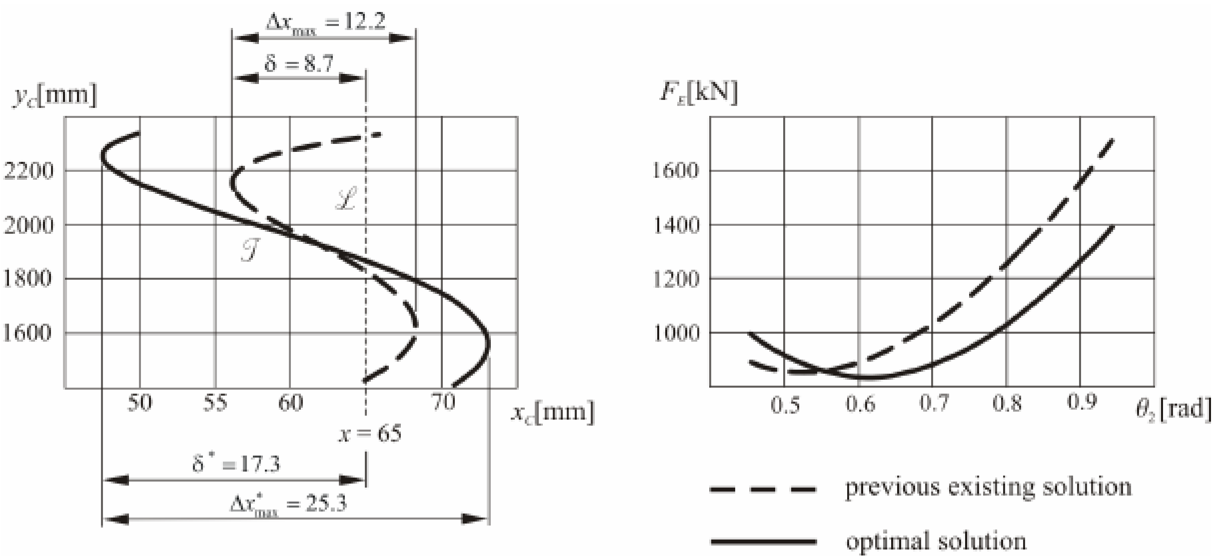

Figure 3. Trajectory of point C and hinge force F_E

The example shows, that with the global optimization approach it is possible to obtain acceptable design of four-bar mechanism, so that hinge forces are minimized, while the difference between the prescribed trajectory and the trajectory of an optionally chosen point C on the coupler remains within the acceptable range. Using the proposed methodology one could obtain the optimal design without exercising mechanical model for various parameter values, while some other design features could be incorporated into formulation at any time.

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